THE STRUCTURE OF COMPLETE MANIFOLDS OF NONNEGATIVE CURVATURE

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0. In this paper we describe some results on the structure of complete manifolds of nonnegative sectional curvature. (We will denote such manifolds by M.) Details and related results will appear elsewhere. Our work generalizes that of Gromoll, Meyer [2] as well as Toponogov [4].

Our analysis is divided into two parts. The first, which we feel is of particular interest, essentially reduces the study of the topology of complete manifolds of nonnegative curvature to that of compact manifolds of nonnegative curvature. The second step reduces the compact case to the compact simply connected case (modulo the classification of compact flat manifolds).

Our results have various applications. One of these is the classification up to isometry of complete 3-dimensional manifolds of nonnegative curvature.

1. The basic notion we use is that of a totally convex set (t.c.s.).

DEFINITION 1.1. A subset C of a riemannian manifold will be called totally convex if for any p, $q \in C$ and any geodesic γ from p to q, we have $\gamma \subset C$.

The significance of total convexity lies in the fact that, topologically, a t.c.s. is very similar to the manifold which contains it. Clearly, any riemannian manifold is a totally convex subset of itself. On the other hand, a point is not totally convex unless M is contractible. In general, nontrivial totally convex sets do not exist. However, in [2] a "basic construction" is given which associates to each point p in a complete noncompact manifold of positive curvature, a compact t.c.s. $C(p)$ containing p. Our fundamental observation is that by use of a different argument involving Toponogov's theorem on geodesic triangles, this construction may be carried out in the case where the curvature of M is nonnegative. More generally we have:

LEMMA 1.1.

(1) *A closed t.c.s. is a totally geodesic submanifold (whose boundary*

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