A STONE-WEIERSTRASS THEOREM FOR SEMIGROUPS[1]

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We prove here the following theorem. We shall assume throughout that the semigroups we consider are commutative and Hausdorff and that they have an identity (this last restriction can be relaxed in some places). By a *semicharacter* of a semigroup G we shall mean a nonzero continuous homomorphism of G into the set of complex numbers with absolute value less than or equal to 1. The product of any two semicharacters of G is not identically zero (since G has an identity) and is therefore a semicharacter. For any semigroup G we shall denote by $\hat{G}$ the set of all semicharacters of G and by G^* the subset of those semicharacters whose absolute value at any point is either 0 or 1.

THEOREM. *If G is a compact semigroup such that G^* separates the points of G, then any subset A of G^* which separates the points of G, which contains the unit semicharacter and which is closed under multiplication and under complex conjugation has to be the whole of G^*.*

As immediate consequences we derive the known theorem on the duality of discrete semigroups which are unions of groups [1] and a theorem on the extendability of semicharacters on compact subsemigroups of a topological semigroup to the whole semigroup. Some other consequences of the theorem will be discussed elsewhere by us. We may remark that a similar theorem was announced as a footnote by Schwarz in [3] with $\hat{G}$ replacing G^* in the above, but it must have been imprecisely stated because it seems to be false for the semigroup $G=[0, 1]$ with the usual multiplication and usual topology.

PROOF OF THE THEOREM. The set of all finite linear combinations of members of A is a separating subalgebra closed under conjugation and containing constants, in the algebra $C(G)$ of all continuous complex valued functions on G and is therefore uniformly dense in $C(G)$ by the Stone-Weierstrass theorem. Thus for every member α of G^*, there are distinct members $\alpha_1, \alpha_2, \cdots, \alpha_k$ in A and complex constants $c_1, c_2, \cdots, c_k$ such that $\left\|\alpha-\sum_{i=1}^{k} c_i\alpha_i\right\|_\infty<1$. If G were a group, a simple argument involving integration with respect to the normalized Haar measure on G would show that $\alpha=\alpha_i$ for some i and in particular, $\alpha\in A$. In the case of a semigroup G we shall apply this argument

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