

ON A PROJECTIVE REPRESENTATION OF THE HALL-JANKO GROUP

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In the line of an investigation of the projective groups of degree six with complex coefficients, it was discovered that the Hall-Janko group, the unique simple group of order 604,800, described in a paper by Marshall Hall and David Wales, has a projective representation of degree six. In fact, there exists a unique group, G_1 , with center, Z , of order two, and $G_1/Z \cong G$, satisfying the following:

(A) the Sylow-7-subgroup of G is normalized by an element of order four;

(B) the inverse image, under the homomorphism $G_1 \rightarrow G_1/Z \cong G$, of a subgroup, $U_3(3)$, of G of order 6048 is isomorphic to $Z \times U_3(3)$. G_1 has two conjugate, faithful, irreducible representations of degree six in the complex field. This representation can be written in $Q(\sqrt{5}, \sqrt{-7})$.

The character table of G_1 can be given, and it is unique. The degrees of the faithful, irreducible representations of G_1 are 6, 6, 64, 64, 50, 50, 216, 14, 84, 126, 126, 252, 56, 56, 448, 350, 336.

Existence of G_1 was verified by taking (mod 3) a representation of degree six. This modular representation restricted to $Z U_3(3)$ has a three dimensional invariant subspace, V . Then, under G_1 , V has one hundred images, which generators of G_1 permute exactly as their images permute the letters described in the Hall-Wales paper. This was checked by computer with a program written by George Shapiro.

Unique unitary matrices over the complex field were obtained for a six dimensional representation of generators of G_1 after the normalizer of a Sylow-7-subgroup was written in a normal form.

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