ON THE GROWTH OF $f(g)$

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It is well known [1] that if f and g are transcendental and entire, then

$$T(r, f(g))/T(r, f) \to \infty \qquad \text{as } r \to \infty. \tag{1}$$

It is reasonable to conjecture that (1) remains valid when f is assumed to be meromorphic instead of entire. (Here and in the sequel it is assumed that the reader is familiar with the Nevanlinna functions $T(r, f)$, $N(r, f)$, $m(r, f)$, etc.)

Using some results of Edrei and Fuchs [2] one can easily verify for any given $\epsilon > 0$ that

$$T(r, f(g)) > \frac{1-\epsilon}{3} T(r, f) \tag{2}$$

for sufficiently large r and for certain families of functions $\{f\}$ and $\{g\}$. For example (2) holds when g is transcendental of finite order and f is transcendental and meromorphic with at least two zeros.

One can also derive other weak results of this type out of Nevanlinna's second fundamental theorem. It seems, however, that anything stronger must be derived from something somewhat more precise than Nevanlinna's theorem.

In this note we show how an extension of the second fundamental theorem can be used to prove (1) for a large class of meromorphic functions g.

Nevanlinna's theorem can be made more precise [3] as follows:

THEOREM 1. *Suppose $f(z)$ is a nonconstant meromorphic function with $f(0) \neq 0, \infty$. For any sequence $a_1, a_2, \cdots, |a_i| \leqq |a_{i+1}|$, with $a_i \neq f(0)$, let $\delta(k) =$ minimum of the distances between the first k points of the sequence. Then for every $k \geqq 2$,*

$$\begin{aligned}(k-1)T(r,f) \leqq{} & N(r,f) + \sum_{i=1}^{k} N\left(r, \frac{1}{f-a_i}\right) \\ & - \left\{2N(r,f) + N\left(r, \frac{1}{f'}\right) - N(r,f')\right\} \\ & + c'k \log r + r(k-1) \log (1/\delta(k)) \\ & + O(\log rT(r, f))\end{aligned} \tag{3}$$