C^*-ALGEBRAS OF TRANSLATIONS AND MULTIPLIERS[1]

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1. **Introduction.** In this note we announce several results about C^*-algebras generated by multiplication and translation operators on L^2-spaces of compact abelian topological groups. The main result, for which the proof is indicated, is that such algebras contain no nontrivial compact operators. It follows that no irreducible, separable C^*-subalgebras of such an algebra can be Type I [2]. We also point out that there are *-isomorphisms between such C^*-algebras on the circle and related C^*-algebras of weighted shifts.

2. **Main result.** Let G be a compact abelian topological group with normalized Haar measure $d\nu$ and consider the associated complex Banach spaces $L^1(G)$, $L^2(G)$, $L^\infty(G)$ and the corresponding real Banach spaces of real-valued functions $L^1_R(G)$, $L^2_R(G)$, $L^\infty_R(G)$. For a in G, an operator T_a is defined on $L^2(G)$ by

$$(T_af)(x) = f(xa).$$

For $\phi(x)$ in $L^\infty(G)$ we can define an operator M_ϕ on $L^2(G)$ by

$$(M_\phi f)(x) = \phi(x)\cdot f(x).$$

We denote by $\tau(G)$ the C^*-algebra generated by all T_a and M_ϕ.

LEMMA 1. *Suppose that for $M>0$ and ϕ_n in $L^\infty(G)$, $1\leqq n\leqq k$, there are $a_i^{(n)}$ in G and real $c_i^{(n)}\geqq 0$ with $1\leqq i\leqq m(n)$, $\sum_{i=1}^{m(n)} c_i^{(n)}=1$ and*

$$\left|\sum_{i=1}^{m(n)} c_i^{(n)}\phi_n(xa_i^{(n)})\right| < M$$

for almost all x in G. Then there are real $c_j\geqq 0$ and a_j in G such that $\sum_{i=1}^{m} c_i=1$ and

$$\left|\sum_{j=1}^{m} c_j\phi_n(xa_j)\right| < M$$

for all $1\leqq n\leqq k$ and almost all x.

PROOF. Let j range over all multi-indices $j=(i_1, i_2, \cdots, i_k)$ where $1\leqq i_n\leqq m(n)$. Then taking

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