AN AXIOMATIC APPROACH TO THE BOUNDARY THEORIES OF WIENER AND ROYDEN

BY PETER A. LOEB AND BERTRAM WALSH[1]

Communicated by John W. Green, April 8, 1968

In this note we announce results, obtained in the framework of Brelot's axiomatic potential theory, which are applicable to the Wiener and Royden boundary theories for Riemann surfaces.[2] Recall that in Brelot's theory, we consider a sheaf $\mathcal{H}$ of real-valued functions with open domains contained in a locally compact, noncompact, connected and locally connected Hausdorff space W, with the functions satisfying certain axioms. Specifically, by a harmonic class of functions on W we mean a class $\mathcal{H}$ of real-valued continuous functions with open domains. For each open $\Omega \subseteq W$, $\mathcal{H}_\Omega$ denotes the set of functions in $\mathcal{H}$ with domains equal to Ω; it is assumed that $\mathcal{H}_\Omega$ is a real vector space. The three axioms of Brelot which $\mathcal{H}$ is assumed to satisfy are (1) a function is in $\mathcal{H}$ if and only if it is locally in $\mathcal{H}$; (2) there is a base for the topology of W which consists of regions regular for $\mathcal{H}$, i.e. connected open sets ω such that any continuous function f on $\partial\omega$ has a unique continuous extension in $\mathcal{H}_\omega$ which is nonnegative if f is nonnegative; (3) the upper envelope of any increasing sequence of functions in $\mathcal{H}_\Omega$ where Ω is a region (i.e. open and connected) is either $+\infty$ or an element of $\mathcal{H}_\Omega$.

Let $\mathcal{H}^-$ and $\mathcal{H}_-$ denote the classes of functions which are superharmonic and subharmonic with respect to $\mathcal{H}$; let $\mathcal{H}^{-b}$ denote the subclass of $\mathcal{H}^-$ consisting of functions bounded below. We assume as another axiom: (4) $\mathbf{1} \in \mathcal{H}_W^-$.

1. Let $\overline{W}$ be a Hausdorff space in which W is imbedded as a dense (and therefore open) subspace, and henceforth let us agree that $\overline{\Omega}$ will mean the closure of Ω in $\overline{W}$ and $\partial\Omega = \overline{\Omega} - \Omega$. If Ω is an open subset of W, we shall say that $\partial\Omega$ *is associated with* $\mathcal{H}_\Omega^{-b}$ if every $v \in \mathcal{H}_\Omega^{-b}$ whose limit inferior is nonnegative at every point of $\partial\Omega$ is necessarily nonnegative on Ω. Throughout this note, we shall denote $\lim_{x\in\Omega, x\to x_0} f(x)$ by $\lim_\Omega f(x_0)$; similar notation is used for lim inf and lim sup.

THEOREM 1.1. *If Ω is an open subset of W and ∂W is associated with $\mathcal{H}_W^{-b}$, then $\partial\Omega$ is associated with $\mathcal{H}_\Omega^{-b}$.*

[1] The first author was supported by National Science Foundation research grants GP-5279 and GP-4653; the second author by GP-4563.

[2] These results will appear with proofs as part of a forthcoming article in the Annales de l'Institut Fourier.