ON THE SPACE OF RIEMANNIAN METRICS[1]

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1. Results. In the present announcement we are concerned with the space of Riemannian metrics on a compact smooth manifold. Let M be such a manifold, S^2T^* the bundle of symmetric covariant two-tensors on M, and $C^\infty(S^2T^*)$ the smooth sections of this bundle, endowed with the C^∞ topology. If $\mathfrak{M}\subseteq C^\infty(S^2T^*)$ is the set of smooth Riemannian metrics on M (those sections which at each point p of M induce a positive definite bilinear form on T_p, the tangent space to M), it is well known that $\mathfrak{M}$ is an open convex cone in $C^\infty(S^2T^*)$. If $\mathfrak{D}$ is the group of diffeomorphisms of M (with the C^∞ topology), $\mathfrak{D}$ acts on $C^\infty(S^2T^*)$ on the right by "pull-back" and $\mathfrak{M}$ is an invariant set under the action. We write $A: \mathfrak{D}\times C^\infty(S^2T^*)\to C^\infty(S^2T^*)$ and denote $A(\eta, \gamma)$ by $\eta^*(\gamma)$. A is a right action because $(\xi\eta)^*\gamma=\eta^*\xi^*(\gamma)$.

Now restrict to $A: \mathfrak{D}\times\mathfrak{M}\to\mathfrak{M}$. For any $\lambda\in\mathfrak{M}$ define I_λ, the isotropy group of λ, by $I_\lambda=\{\eta\in\mathfrak{D}\mid\eta^*(\lambda)=\lambda\}$. For a fixed $\gamma\in\mathfrak{M}$, let O_γ be the orbit of $\mathfrak{D}$ through γ.

MAIN THEOREM.

(1) *A induces a homeomorphism of* $\mathfrak{D}/I_\gamma$ *onto* O_γ *by* $\eta I_\gamma\to\eta^*(\gamma)$.

(2) *There is a subspace S of* $\mathfrak{M}$ *containing* γ *which has the following properties*:

(a) $A(I_\gamma, S)=S$,

(b) *If* $\eta\in\mathfrak{D}$ *and* $\eta^*(S)\cap S\neq\emptyset$, *then* $\eta\in I_\gamma$,

(c) *There exists a neighborhood U of the identity coset in* $\mathfrak{D}/I_\gamma$ *and a local cross section* χ: $U\to\mathfrak{D}$ *such that the map* F: $U\times S\to\mathfrak{M}$ *by* $F(u, s)=(\chi(u))^*(s)$ *is a homeomorphism onto a neighborhood of* γ.

S is called a slice through γ. From this theorem it follows easily that for any $\lambda\in\mathfrak{M}$ which is sufficiently near γ, there exists $\eta\in\mathfrak{D}$ such that $I_\lambda\subseteq\eta I_\gamma\eta^{-1}$.

I_γ is by definition the group of isometries of $\mathfrak{M}$ with respect to the metric γ, so we have shown that this group cannot increase locally. In particular we know that the set $\mathcal{G}$ of metrics which have trivial

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