# BOUNDARY VALUE PROBLEMS FOR DELAY-DIFFERENTIAL EQUATIONS

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1. **Introduction.** In this note we shall give some sufficient conditions for the existence of solutions of a certain type of boundary value problem (BVP) for delay-differential equations (d.d.e.'s). The conditions given are of two kinds, in Theorem 1 a relationship between the boundary conditions and the size of the interval under consideration implies the existence of solutions; in Theorem 4 the existence of solutions of delay-differential inequalities implies the existence of solutions. A discussion concerning the formulation of BVP's of the type considered here may be found in [1], [2], and [3]; these sources in turn reveal much of the literature concerning such problems.

2. **The problem.** Let $f$ be a real-valued continuous function defined on $R^{n+m+2}\times I$, where $I$ is the compact interval $[a, b]$. Let $h_1(t), \cdots, h_n(t), g_1(t), \cdots, g_m(t)$ be nonnegative continuous functions with domain $I$. Assume that $t-g_i(t)$ assumes the value $a$ at most a finite number of times as $t$ ranges over $I$ and $i=1, \cdots, m$. Define the real number $c$ by

$$c = \min \left\{ \min_{1\leqq i\leqq n} \inf_{t\in I} (t - h_i(t)), \min_{1\leqq j\leqq m} \inf_{t\in I} (t - g_j(t)) \right\}$$

and let $J=[c, a]$. Let $\phi(t)\in C^1(J)$ and let $B$ be any real number; we then seek a function $x(t)\in C(J\cup I)\cap C^1(J)\cap C^1(I)$ having a piecewise continuous second derivative such that

$$(1)\quad x(t) = \phi(t), \qquad x'(t) = \phi'(t), \qquad t\in J, \qquad x(\bar{b}) = B, \qquad \bar{b} \leqq b.$$

and

$$(2)\quad x''(t) = f(x(t), x(t-h_1(t)), \cdots, x(t-h_n(t)), x'(t), x'(t-g_1(t)), \cdots, x'(t-g_m(t), t)$$

for $a \leqq t \leqq \bar{b}$.

In general we must expect that a solution of problem (1)–(2) will have a discontinuous derivative at $t=a$, and therefore the second derivative will in general only be piecewise continuous if the right side of (2) depends on delays in $x'$.

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