PERTURBING ASYMPTOTICALLY STABLE DIFFERENTIAL EQUATIONS

BY AARON STRAUSS[1] AND JAMES A. YORKE[2]

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Our purpose here is to announce new theorems on the eventual uniform-asymptotic stability (hereafter called EvUAS) of the origin 0 for the ordinary differential equation

$$\text{(P)} \qquad x' = f(t, x) + g(t, x), \qquad (x' = dx/dt)$$

given that 0 is EvUAS for the equation

$$\text{(E)} \qquad x' = f(t, x),$$

and that f and g satisfy certain conditions. We always assume that f and g are at least continuous from $[0, \infty) \times R^d$ to R^d, but we never assume that the solutions of (P) are unique or that the zero function is a solution of (P). In fact EvUAS is a natural generalization of uniform asymptotic stability in which it is not assumed that the zero function is a solution.

Our main result is (definitions follow)

THEOREM A. *Let* 0 *be EvUAS for* (E). *Then* 0 *is EvUAS for* (P) *if*

(i) *f is Lipschitz and g is diminishing, or*

(ii) *f is periodic and g is diminishing, or*

(iii) *f is inner product and g is absolutely diminishing, or*

(iv) *f is linear and $g = g_1 + g_2$, where g_1 is absolutely diminishing and* $g_2 = o(|x|)$.

Let $x(t; t_0, x_0)$ denote a solution of (E) through (t_0, x_0). We say that 0 is EvUAS for (E) if

$$\lim_{t_0 \to \infty;\, |x_0| \to 0} \left[\sup_{t \geqq t_0} |x(t; t_0, x_0)|\right] = 0$$

and if, for some $\delta_0 > 0$ and some $\alpha_0 \geqq 0$,

$$\lim_{t \to \infty} \left[\sup_{t_0 \geqq \alpha_0;\, |x_0| < \delta_0} |x(t + t_0; t_0, x_0)|\right] = 0.$$

[1] Department of Mathematics, University of Maryland, College Park, Maryland. Research supported in part by a National Science Foundation Postdoctoral Fellowship and in part by NSF grants GP-4921 and GP-6167.

[2] Institute for Fluid Dynamics and Applied Mathematics, University of Maryland, College Park, Maryland. Research supported in part by NSF grants GP-6114 and GP-7846.