ON MORSE-SMALE DIFFEOMORPHISMS

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Let Diff(M) denote the group of C^r diffeomorphisms of a closed C^∞ manifold M endowed with the C^r topology, $r \geqq 1$. In [6] Smale defined a subset of Diff(M), later called Morse-Smale diffeomorphisms, and conjectured that this subset is open and each of its elements is structurally stable.

The main purpose of this note is to announce the proof of this conjecture in the case dim $M \leqq 3$, and to state some related results. These results are also valid for vector fields and then they extend results of [4] proved for the case dim $M = 2$.

1. **Preliminaries.** Following [8], for $f \in$ Diff(M) we denote by Per(f) the set of periodic points and by $\Omega(f)$ the set of nonwandering points of f. A point $x \in$ Per(f) of period n is hyperbolic if the derivative $(Df^n)_x$ has its spectrum disjoint from the unit circle in the complex plane C. In this case we have the existence of stable and unstable manifolds of x, denoted by $W^s(x)$ and $W^u(x)$. We call dim $W^s(x)$ the stable index of x.

DEFINITION 1. $f \in$ Diff(M) is called Morse-Smale if it satisfies the following conditions.

(1) $\Omega(f)$ is finite. This implies $\Omega(f) = $ Per(f).

(2) All points in Per(f) are hyperbolic.

(3) For any pair of points $x, y \in$ Per(f), $W^s(x)$ and $W^u(y)$ are in general position.

In the sense of Smale [8], this means that f satisfies Axiom A and a strong version of Axiom B, $\Omega(f)$ being finite. If f is Morse-Smale, the periodic orbits of f can be partially ordered by the relation: $\mathcal{O}(x) \leqq \mathcal{O}(y)$ if $W^s(x) \cap W^u(y) \neq \emptyset$. The periodic orbits of f with this partial order structure is called the phase-diagram of f, denoted by $D(f)$. By a diagram isomorphism we mean a map ρ: $D(f) \to D(g)$ which is bijective, index and order preserving.

DEFINITION 2. $f \in$ Diff(M) is structurally stable if f has a neighborhood V in Diff(M) such that for any $g \in V$ there exists a homeomorphism h: $M \to M$ satisfying $hf(x) = gh(x)$ for all $x \in M$.

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