A SPECTRAL SEQUENCE FOR CLASSIFYING LIFTINGS IN FIBER SPACES[1]

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Consider the following diagram of pointed spaces and maps

$$\begin{tikzcd} & Y \arrow[d, "p"] \\ X \arrow[ur, "g"] \arrow[r, "f"'] & B \end{tikzcd}$$

where $pg=f$ and p is a fibration with fiber F. Suppose that X is a CW-complex of dimension $\leqq 2\operatorname{conn}(F)$ and $\operatorname{conn}(F)\geqq 1$ (conn $=$connectivity). Let $[X, Y]_B$ be the set of homotopy classes of pointed maps over f(H: $X\times I\to Y$ is a homotopy over f if $pH_t=f$ for each $t\in I$). Becker proved in [2], [3] that under these hypotheses $[X, Y]_B$ can be given an abelian group structure with $[g]$ as zero element.

The purpose of this note is to describe a spectral sequence of the Adams type which converges to $[X, Y]_B$. The differentials of the spectral sequence are the twisted operations described in [6], [7]. The sequence has the same relation to the method of computing $[X, Y]_B$ used in [6], [7] as the Adams spectral sequence has to the killing-homotopy method of computing ordinary homotopy groups. This note should be read as a sequel to [7].

A different spectral sequence for $[X, Y]_B$ is given by Becker in [3]. A sequence apparently similar to the one to be described here is mentioned in [4] and credited to Becker and Milgram.

1. The spectral sequence. Consider the following commutative diagram:

$$\begin{tikzcd} & Y^2 \arrow[r] \arrow[d, shift left, "q"] & Y \arrow[d, "p"] \\ X \arrow[r, "g"] \arrow[rr, bend right=10, "f"'] & Y \arrow[u, shift left, "s"] \arrow[r, "p"] & B \end{tikzcd}$$

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