THE STRUCTURE OF TORSION ABELIAN GROUPS GIVEN BY PRESENTATIONS[1]

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Let F_X denote the free abelian group freely generated by the set X, and let R be a subset of F_X. With $[R]$ denoting the subgroup of F_X generated by R, set

$$G(X, R) = F_X/[R],$$

i.e., $G(X, R)$ is that abelian group generated by X and subject only to the relations

$$r = 0 \qquad \text{all } r \in R.$$

If each of the elements in R involves only one generator in X, then $G(X, R)$ is a direct sum of cyclic groups. On the other hand, if G is any abelian group, then $G \cong G(X, R)$, where each element in R involves at most three generators in X; indeed this isomorphism results if we take $X = G$ and R equal to the set of all elements in F_G of the form $x+y-z$, where $z=x+y$ in G.

Our purpose here is to investigate the structure of the group $G(X, R)$ in the intermediate case when each of the elements of R involves at most two generators, and $G(X, R)$ is a torsion group. We can evidently restrict our attention to p-groups, and in this case it is easily seen that $G(X, R) \cong G(X', R')$, where each element in R' is of one of the forms

$$p^n x \quad \text{or} \quad p^n x - y.$$

This leads us to the following definition. Let X be a set, V be a subset of the set of ordered pairs $\langle x, y\rangle$ with $x, y \in X$, u be a map of X to the nonnegative integers, and v be a map of V to the nonnegative integers. By $G(X, V, u, v)$ we mean that abelian group generated by X and subject only to the relations

$$p^{u(x)}x = 0 \qquad \text{all } x \in X,$$
$$p^{v(x,y)}x = y \qquad \text{all } \langle x, y\rangle \in V.$$

We say that an abelian p-group G is a *T-group* if $G \cong G(X, V, u, v)$ for some $\langle X, V, u, v\rangle$.

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