CELL-LIKE MAPPINGS OF ANR'S

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We introduce here the concept of "cell-like" mappings, i.e. mappings with "cell-like" inverse sets (definition below). For maps of ANR's, this concept is the natural generalization of cellular maps of manifolds (see (3) below). Also, a proper mapping of ANR's is cell-like if, and only if, the restriction to any inverse open set is a proper homotopy equivalence. This latter condition is one studied by Sullivan in connection with the Hauptvermutung (see [8]).

DEFINITION. A *space A is cell-like* if there is an embedding ϕ of A into some manifold M such that $\phi(A)$ is cellular in M (see [3]). A *mapping* $f: X \to Y$ *is cell-like* if $f^{-1}(y)$ is a cell-like space for each $y \in Y$.

The following technical property is useful in studying cell-like spaces.

PROPERTY (**). A map $\phi: A \to X$ has Property (**) if, for each open set U of X containing $\phi(A)$, there is an open set V of X, with $\phi(A) \subset V \subset U$, such that the inclusion $V \subset U$ is null-homotopic (in U).

The above terminology arose in generalizing McMillan's cellularity criterion [6] to hold for cell-like spaces. S. Armentrout [1] has independently studied this property, calling it "property UV^∞".

To avoid confusion, we will assume that an ANR is a retract of a neighborhood of euclidean space $\mathbf{R}^n$.

THEOREM 1. *Let A be a compact, finite-dimensional metric space. Then the following are equivalent*:

(a) *A is cell-like.*

(b) *A has the "fundamental shape" or "Čech homotopy type" of a point, as defined by Borsuk in* [2].

(c) *There exists an embedding of A into some ANR which has Property* (**).

(d) *Any embedding of A into any ANR has Property* (**).

Working independently and from a different point of view, Armentrout has obtained results quite similar to Theorem 1. The proof is not hard. The implications (a)$\Rightarrow$(b)$\Rightarrow$(c)$\Rightarrow$(d) make use only of elementary properties of ANR's; (d)$\Rightarrow$(a) is easy using [6].

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