# A NOTE ON VOLTERRA INTEGRAL EQUATIONS AND TOPOLOGICAL DYNAMICS[1]

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1. **Introduction.** In a recent paper, G. R. Sell [5], [6], has developed methods which allow one to apply the theory of topological dynamics to a very general class of nonautonomous ordinary differential equations. The purpose of this note is to illustrate how the methods of Sell can be extended to nonlinear Volterra integral equations of the form

$$x(t) = f(t) + \int_0^t a(t, s)g(x(s), s)ds. \tag{1}$$

A complete discussion of our results along with the proofs of the theorems noted here will appear in [3] and [4]. In this note we shall restrict ourselves to a description of the semiflow generated by (1), and we do this in the case where $x, f, a$ and $g$ are real-valued.

Because of the generality of our methods, they can be applied to many problems. Some of these applications are treated in [4]. We shall illustrate our techniques by analyzing a problem of J. Levin [1] in §5.

2. **Construction of the semiflow.** A *flow* is defined to be a mapping $\pi: X\times R\to X$, where $X$ is a topological space and $R$ the reals, that satisfies (i) $\pi(x, 0)=x$, (ii) $\pi(\pi(x, t), s)=\pi(x, t+s)$ and (iii) $\pi$ is continuous. A (local) flow was defined in [5], and for this note we need the concept of a (local) semiflow, in which we restrict $t$ to be nonnegative. A local flow differs from a flow in the sense that motions $\pi(x, t)$ may fail to exist for all time $t$. We refer the reader to [5] and [7] for details.

For Eq. (1), the semiflow is constructed as follows: Let $\phi(t) =\phi(f, g, a; t)$ denote the solution of (1). Under hypotheses on $f$, $g$ and $a$ which are stated below, it is shown in [3], that $\phi$ is uniquely determined and depends continuously on $f$, $g$, $a$ and $t$. Now define the function $T_\tau f = T_\tau(f, g, a)$ by

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