ON SIMPLE GROUPS OF ORDER $5\cdot 3^a\cdot 2^b$

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The following theorem can be proved.

THEOREM. *If G is a simple group of an order g of the form $g=5\cdot 3^a\cdot 2^b$, $g\neq 5$, then G is isomorphic to one of the alternating groups A_5, A_6, or to the group $O_5(3)$ of order* 25,920.

One may conjecture that there exist only finitely many nonisomorphic noncyclic groups whose order g is divisible by exactly three distinct primes $p<q<r$. J. G. Thompson [6] has shown that then $p=2$, $q=3$ while r is 5, 7, 13, or 17. It is not unlikely that if one of the exponents a,b,c is 1, the methods applied here can be used to find all simple groups of the orders in question. No example is known in which all three exponents a,b,c are larger than 1.

Since the proof of the theorem is long, we do not intend to publish it. A complete account has been prepared in mimeographed form.[2] We shall give a brief outline.

1. We start with two propositions of slightly more general interest.

PROPOSITION 1. *Let G be a simple group of an order $g=p^aq^br^c$ where p,q,r are distinct primes. Assume that the Sylow-subgroup R of G of order r^c is cyclic. Then R is self-centralizing in G; $C(R)=R$.*

PROOF. If this was false, we may assume that $C(R)$ contains an element π of order p, (interchanging p and q, if necessary). Then, for $R=\langle\rho\rangle$,

$$\sum \chi_j(\pi\rho)\chi_j(1) = 0$$

where χ_j ranges over the irreducible characters of G in the principal p-block $B_0(p)$. It follows that there exists a nonprincipal character $\chi_j\in B_0(p)$ such that

$$(1)\qquad \chi_j(1)\not\equiv 0 \pmod q,\qquad \chi_j(\pi\rho)\neq 0.$$

If here χ_j belongs to the r-block $B(r)$, the second condition (1) implies that ρ belongs to a defect group D of $B(r)$, cf. [2]. Thus, $D=R$. It

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[2] This report can be obtained on request from the Department of Mathematics, Harvard University, Cambridge, Massachusetts 02138.