# NORMS AND THE LOCALIZATION OF ROOTS OF MATRICES[1]

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Before I embark upon my topic, perhaps a bit of orientation might be in order. The topic lies in a borderland between numerical analysis and pure mathematics. The problems arise out of very important practical computational difficulties; the results are due almost exclusively to those who have rather more than a passing interest in computation; they have been published largely in books and journals devoted to numerical analysis and applied mathematics; but the more elaborate developments go rather beyond anything that can promise any direct and immediate practical application. Here we enter what I think must be classified as pure mathematics.

The phrase "numerical analysis" itself is a fairly recent coinage, dating back about twenty years to the advent of the electronic computer. What it refers to is neither pure mathematics nor classical applied mathematics. I would say that classical applied mathematics is primarily descriptive, at least in intent, whereas numerical analysis is primarily prescriptive in intent. To the extent that it is descriptive, it describes, or attempts to describe, processes and their outcome, but in terms that permit a rational choice among possibly competing alternatives. I leave you to characterize pure mathematics as you like, but I think it differs, at least in intent, from either. I make these points only for purposes of orientation and not argumentation.

My title is in two parts and is intended to indicate an intersection and not a union. A localization theorem for roots of a matrix is one that gives information concerning the location of the roots. There are two main classes, exclusion and inclusion. An inclusion theorem designates a set in the complex plane containing at least one root of a given matrix. An exclusion theorem designates a set containing none, so that its complement contains all. It is easy to see why such theorems would be of interest to numerical analysts. Any computation that does not employ exact arithmetic, and this means most, can produce only an approximation, at best, to the mathematically defined result. Naturally one is therefore interested in knowing the

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