MORE GROUPS THAT ARE JUST ABOUT FREE

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1. A residually nilpotent group G is termed *parafree of rank r* if $G/\gamma_i G \cong F/\gamma_i F$ for $i=1, 2, \cdots$, where F is a free group of rank r and $\gamma_i X$ is the ith term of the lower central series of the group X. The object of this announcement is to make known the existence of certain nonfree parafree groups with the most extraordinary properties. Unlike the groups in [1] and [2] these groups are not finitely generated, but they have even more surprising properties.

THEOREM. *Let $r>1$ be an integer and let $V_1, V_2, \cdots$ be any countable family of varieties, none of which is the variety of all groups. Then there exists a parafree group G of rank r such that*

(i) *G is locally free;*

(ii) *$G/V_i(G) \cong F/V_i(F)$ $(i=1, 2, \cdots)$ (where $V_i(X)$ is the least normal subgroup of the group X with factor group $X/V_i(X)$ in the variety V_i);*

(iii) *G is not free.*

2. At present it is not known whether there are more than a countably infinite number of varieties. Thus we may take

$$V_1, V_2, \cdots$$

to be the set of all known varieties of groups, excluding the variety of all groups. So this set will contain for each finite group the variety generated by it, and also the variety S_i generated by all soluble groups of derived length at most i. The resultant locally free, parafree group G provided by the theorem will therefore have, in particular, the following properties:

(a) G has precisely the same number of subgroups of finite index j as the free group F of rank r for $j=1, 2, \cdots$;

(b) every finite homomorphic image of G can be generated by k elements;

(c) $G/\delta_i G \cong F/\delta_i F$ for $i=1, 2, \cdots$ (where $\delta_i X$ is the ith derived group of X).

3. The groups G of the theorem are constructed as a colimit (i.e. direct limit) of free groups of rank r in a very simple way.

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