BASIC SETS OF INVARIANTS FOR FINITE REFLECTION GROUPS

BY LEOPOLD FLATTO[1]

Communicated by J. K. Moser, February 21, 1968

1. **Introduction.** Let V be an n-dimensional vector space over a field K of characteristic zero. Let G be a finite group of linear transformations of V. G acts naturally as a group of automorphisms of the ring of polynomials $K[x]$ if we define $(gP)(x)=P(g^{-1}x)$ for $g\in G$, $P(x)\in K[x]$. The polynomials which are invariant under G form an algebra I over K called the algebra of invariants of G. A linear transformation is said to be a reflection if it has finite order and leaves fixed an $(n-1)$-dimensional hyperplane, called its reflecting hyperplane. G is a finite reflection group if it is of finite order and is generated by reflections. Chevalley [5] has proved that for finite reflection groups, I has an integrity basis consisting of n algebraically independent forms $I_1, \cdots, I_n$. Furthermore, Shephard and Todd [10] have shown that this property of I characterizes the finite reflection groups.

If K is the real field R, then G leaves invariant a positive definite quadratic form [2] so that G is orthogonal after a linear change of variables. Coxeter [3], [4] has classified all irreducible finite orthogonal reflection groups and has computed the degrees $m_1, \cdots, m_n$ of the forms $I_1, I_2, \cdots, I_n$. These degrees are independent of the particularly chosen basis. We provide a method for computing an explicit integrity basis of I for these groups. We will relate this problem to a certain mean value problem.

2. **Construction of the basic set of invariants.** We now state several theorems and sketch some of the proofs. Full details will appear elsewhere. Our first theorem yields a formula for the product of homogeneous invariants forming an integrity basis of I.

THEOREM 2.1. *Let G be an irreducible finite orthogonal reflection group acting on the real n-dimensional space E_n. Let $P_m(x, y) = \sum_{\sigma\in G}(x\cdot\sigma y)^m$ $(1\leqq m<\infty)$, where $x\cdot y=x_1y_1+\cdots+x_ny_n$. Let $J(x, y)=\partial(P_{m_1}, \cdots, P_{m_n})/\partial(x_1, \cdots, x_n)$, where $m_1, \cdots, m_n$ denote the respective degrees of the basic invariant forms $I_1, \cdots, I_n$. Then $J(x, y)=\prod_{i=1}^{n} J_i(y)\prod_{i=1}^{r} L_i(x)$. The J_i's are homogeneous invariants*

[1] Supported by National Science Foundation Grant GP-7352.