ON THE SYMBOL OF A PSEUDO-DIFFERENTIAL OPERATOR

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In [1] Hörmander defines the generalized symbol of a pseudo-differential operator P as a sequence of partially defined maps between function spaces. Our purpose here is to comment on the existence of characteristic polynomial type symbols $\sigma(P)$ and to obtain their composition by introducing a product structure on suitable jet bundles. In particular, this gives the lower order symbol for differential operator on manifold. I express my hearty thanks to J. Bokobza, H. Levine, and A. Unterberger for their indispensable help.

1. **Operation in jet bundle.** Given a compact C^∞ differentiable manifold X, we denote by

$$p_k:J^m(\boldsymbol{R}) \to J^k(\boldsymbol{R}), \qquad m \geqq k,$$

the jet bundle of the trivial bundle $X\times \boldsymbol{R}$ and the canonical projection. Identify the cotangent bundle $T(X)$ as a subbundle of $J^1(\boldsymbol{R})$ we define the subbundle

$$J_0^k(\boldsymbol{R}) \subseteq J^k(\boldsymbol{R}), \qquad k \geqq 1,$$

as the inverse image by p_1: $J^k(\boldsymbol{R})\to J^1(\boldsymbol{R})$ of the nonzero cotangent vector $T_0(X)\subseteq T(X)$. Let E, F, and G be complex vector bundles over X and put

$$J^*(E, F) = \prod_{k=0} \operatorname{Hom}(J_0^{k+1}(\boldsymbol{R}) \oplus J^k(E), F)$$

where "Hom" denotes the space of C^∞ bundle maps which are linear with respect to $J^k(E)$. We shall construct an operation

$$\circ:J^*(E, F) \times J^*(F, G) \to J^*(E, G)$$

as follows. If $\alpha = (\alpha_0, \alpha_1, \cdots, \alpha_m, \cdots) \in J^*(E, F)$, $\beta = (\beta_0, \beta_1, \cdots) \in J^*(F, G)$, then

$$\alpha \circ \beta = (\gamma_0, \gamma_1, \cdots, \gamma_r, \cdots) \in J^*(E, G)$$

is given by

$$\gamma_r = \sum_{m+n=r} \beta_n \circ (p_{n+1} \circ p_R \oplus j^n(\alpha_m))$$

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