SOME RESULTS ON ONE-RELATOR GROUPS

BY B. B. NEWMAN

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The results in this note arose from considering the question: What are the abelian subgroups of a one-relator group? The additive group of p-adic rationals and the free abelian group of rank 2 are certainly subgroups of a one-relator group. For example in

$$G = \mathrm{gp}\{a, b; a^{-1}b^{2p}ab^{-2}\}$$

the infinitely generated subgroup

$$H = \mathrm{sgp}\{b^2, a^{-1}b^2a, a^{-2}b^2a^2, \cdots\}$$

is isomorphic to the additive group of p-adic rationals, and

$$K = \mathrm{sgp}\{b^2, b^{-1}a^{-1}ba\}$$

is free abelian of rank 2. In 1964 Gilbert Baumslag [1] conjectured that the additive group of rationals is not a subgroup of a one-relator group. That this conjecture is correct follows from the following theorem.

THEOREM 1. *Let G be a torsion-free one-relator group. Then no nontrivial element of G has more than finitely many prime divisors. Moreover a nontrivial element is not divisible by more than finitely many powers of a prime p if p is greater than the length of the relator.*

REMARK. An element g of a group is divisible by an integer n, or has a divisor n, if g has an nth root in the group. By the length of the relator is meant the letter length of the relator as a word in a free group.

R. C. Lyndon [7] has shown that the cohomological dimension of a torsion-free one-relator group is $\leqq 2$. Now the cohomological dimension of a free abelian group of rank n is n, and of a direct product of an infinite cyclic group with a noncyclic locally cyclic group is >2, (see [6], [2]). Since the cohomological dimension of a subgroup is less than or equal to the cohomological dimension of the group, it follows that the only abelian subgroups of a torsion-free one-relator group are free abelian of rank $\leqq 2$ or locally cyclic subgroups in which every nontrivial element is divisible by at most finitely many primes.

The proof of Theorem 1 uses the usual argument of the Freiheitssatz (see [9]) together with the following ideas.

DEFINITION. Let H be a subgroup of G and p a prime. Then H is