ALMOST EVERYWHERE CONVERGENCE OF POISSON INTEGRALS ON GENERALIZED HALF-PLANES

BY NORMAN J. WEISS[1]

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1. **Introduction.** A classical theorem of Fatou states that if f is an L^p function on the line (circle), $p \geqq 1$, and if the harmonic function F on the upper half-plane (disk) is the Poisson integral of f, then $F(z) \to f(x)$ as $z \to x$ nontangentially for a.e. x on the line (circle).

Generalizations in several directions have recently been found, e.g. [1], [2], [4], [6]. Our result, stated precisely below, is Fatou's theorem for generalized upper half-planes holomorphically equivalent to bounded symmetric domains and functions of type L^p, $p>1$, or locally of type $L \log {}^+L$. Details will appear elsewhere.

In §2, we sketch the setting and state our result explicitly. The proof is case-by-case, and includes the case of the exceptional domains; §3 is devoted to a sketch of the proof in a typical case.

2. **The theorem.** Let D be a generalized upper half-plane, i.e.

$$D = \{(z, w) \in V_1 \times V_2 \colon \operatorname{Im} z - \Phi(w, w) \in \Omega\},$$

where V_1 is a complex vector space with a given real form, V_2 is a complex vector space, $\Omega \subset \operatorname{Re} V_1$ is an open cone, and $\Phi\colon V_2 \times V_2 \to V_1$ is hermitian symmetric bilinear with respect to $\operatorname{Re} V_1$ such that $\Phi(w, w) \in \bar{\Omega}$. When Ω is a domain of positivity and Φ satisfies certain symmetry and homogeneity properties, D is holomorphically equivalent to a bounded symmetric domain [5]. The distinguished boundary of D is

$$B = \{(z, w) \colon \operatorname{Im} z - \Phi(w, w) = 0\}.$$

We identify B with $\operatorname{Re} V_1 \times V_2$ by associating to $(x + i\Phi(w, w), w)$ the pair (x, w). There is a nilpotent group $\mathfrak{N}$ of automorphisms of D which acts transitively on B and is also equal to $\operatorname{Re} V_1 \times V_2$ as a set. Haar measure on $\mathfrak{N}$ is the induced Euclidean measure.

The Poisson kernel, $P(u, \zeta)$, is defined on $B \times D$, and the Poisson integral of a function f on B is

$$F(\zeta) = \int_B f(u) P(u, \zeta) du.$$

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