SOME REMARKS ON $l-l$ SUMMABILITY

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Introduction. If x is a complex number sequence and $A=(a_{nk})$ is an infinite matrix of complex numbers, then A determines a transformation of x into the sequence Ax, where $(Ax)_n=\sum_k a_{nk}x_k$. Let l represent the set of complex sequences with finite norm $\|x\|=\sum|x_k|$. If $Ax\in l$ whenever $x\in l$, then A is called an $l-l$ method of summation. Let l_A denote the summability field of A, that is, the set of all sequences x such that $Ax\in l$. In [3] an attempt is made to characterize those $l-l$ methods A for which $l_A=l$. In what follows we give counterexamples to Theorems 7 and 9 of [3], announce several positive results related to these theorems, and generalize Theorem 5 of [3] for the class of factorable $l-l$ methods.

1. Terminology and notation. Knopp and Lorentz [4] show that the matrix A is an $l-l$ method if and only if $\|A\|<\infty$, where $\|A\|=\sup_k \sum_n|a_{nk}|$. $\|A\|$ is the norm of A as an operator from l to l. It is known that l_A inherits a locally convex topology making it an FK space. Moreover, each $f\in l_A'$, the dual space of l_A, has the representation

$$f(x)=\sum_n t_n\sum_k a_{nk}x_k+\sum_k \beta_k x_k$$

for some bounded sequences t and β. An $l-l$ method A is reversible if the equation $y=Ax$ has a unique solution x in l_A for each y in l. An $l-l$ method A is called perfect if l is dense in l_A (equivalently, if the set $\Delta=\{e^k: k=1, 2, \cdots\}$, where e^k is the sequence having a one in the kth coordinate and zeros elsewhere, is fundamental in l_A). Let m_r denote the set of all sequences x with finite norm $\|x\|=\sup_m|\sum_{k=1}^m x_k|$. In [3], an $l-l$ method A is called 0-perfect if every sequence x in $m_r\cap l_A$ is a limit point in l_A of the set l. Concerning these concepts, Jürimäe makes the following two statements. (In Statement B he omits the assumption of reversibility.)

STATEMENT A [3, THEOREM 7]. If A is an 0-perfect $l-l$ method with $l_A\subseteq m_r$, then $l_A=l$.

STATEMENT B [3, THEOREM 9]. A reversible 0-perfect $l-l$ method B sums a sequence $x\notin l$ if and only if

$$\sup_m \sup_k\left|\sum_{n=1}^m b'_{nk}\right|=\infty, \tag{1}$$