SPHERICAL FUNCTIONS ON A $\mathfrak{p}$-ADIC CHEVALLEY GROUP

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1. Introduction. In [6] Satake set up a general theory of zonal spherical functions on a reductive linear algebraic group over a $\mathfrak{p}$-adic field. One problem left open was the determination of the explicit form of the spherical functions and the Plancherel measure. In this note we shall present explicit formulas for these things in the case of a Chevalley group. Detailed proofs will appear elsewhere.

2. **Notation.** Let $\mathfrak{o}$ be a complete discrete valuation ring and K its field of fractions. Choose a generator π of the maximal ideal of $\mathfrak{o}$. Suppose that the residue field of $\mathfrak{o}$ is *finite* and has q elements. Then K is locally compact (and totally disconnected) with respect to the valuation topology.

Let $\mathfrak{g}$ be a complex semi-simple Lie algebra, $\mathfrak{h}$ a Cartan sub-algebra of $\mathfrak{g}$, and V the vector-space dual of $\mathfrak{h}$. The dual V^* of V is then identified with $\mathfrak{h}$. If $\xi \in \mathfrak{h} = V^*$ and $\eta \in V$ we write (ξ, η) for the value of η at ξ (or of ξ at η). Let $\Delta \subset V$ be the set of (nonzero) roots of $\mathfrak{g}$ relative to $\mathfrak{h}$, and for each root α let $\alpha^* \in V^*$ be the corresponding coweight. Choose a system of fundamental roots Π in Δ; we can then speak of positive and negative roots. The roots generate a free abelian group $R \subset V$ of rank $l = \dim V$, and Π is a basis of R. Let R^* denote the set of all $\xi \in V^*$ such that $(\xi, \alpha) \in \boldsymbol{Z}$ for all roots α. Then $R^* \cong \mathrm{Hom}(R, \boldsymbol{Z})$ and is a free abelian group of rank l contained in V^*, and contains all the coweights α^*. Let R^*_+ be the set of all $\lambda \in R^*$ such that $(\lambda, \alpha) \geqq 0$ for all positive roots α.

The Weyl group W of $\mathfrak{g}$ (relative to $\mathfrak{h}$) acts on $\mathfrak{h}$ and hence on V by transposition. Any element of R^* can be brought into R^*_+ by the action of some element of W. For each $w \in W$ let $n(w)$ denote the number of positive roots α such that $w(\alpha) < 0$, and let $P(t)$ denote the Poincaré polynomial

$$P(t) = \sum t^{n(w)} \tag{2.1}$$

summed over all $w \in W$. More generally, if $\lambda \in R^*$ let

$$P_\lambda(t) = \sum t^{n(w)} \tag{2.2}$$

summed over all $w \in W$ such that $w(\lambda) = \lambda$. Then $P_\lambda(t)$ divides $P(t)$, and we put