NONSOLVABLE FINITE GROUPS ALL OF WHOSE LOCAL SUBGROUPS ARE SOLVABLE[1]

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TABLE OF CONTENTS

1. Introduction. The results of this paper grew from an attempt to classify the minimal simple groups. For obvious reasons, this paper is a natural successor to 0.[2] The structure of the proof showed that a larger class of groups could be mastered with some further effort. An easy corollary classifies the minimal simple groups.

In a broad way, this paper may be thought of as a successful translation of the theory of solvable groups to the theory of simple groups. By this is meant that a substantial structure is constructed which makes it possible to exploit properties of solvable groups to obtain delicate information about the structure and embedding of many solvable subgroups of the simple group under consideration. In this way, routine results about solvable groups acquire great power.

In somewhat more detail, the arguments go as follows, apart from numerous special cases which involve groups of small order: Let $\mathfrak{G}$ be a finite group. Let $\mathcal{S}o\ell(\mathfrak{G})$ be the set of all solvable subgroups of $\mathfrak{G}$. Then $\mathcal{S}o\ell(\mathfrak{G})$ is partially ordered by inclusion and we let $\mathcal{MS}(\mathfrak{G})$ be the set of maximal elements of $\mathcal{S}o\ell(\mathfrak{G})$. Let $\mathcal{M}^*(\mathfrak{G})$ be the set of all elements of $\mathcal{S}o\ell(\mathfrak{G})$ which are contained in precisely one element of

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[2] 0 refers to *Solvability of groups of odd order*, W. Feit and J. Thompson, Pacific J. Math. (3) **13**(1963), and Result X of 0 is here referred to as Result 0.X. Also, as in 0, (B) refers to Theorem B of [26].