

## REFERENCES

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## GLOBAL ASYMPTOTIC ESTIMATES FOR ELLIPTIC SPECTRAL FUNCTIONS AND EIGENVALUES<sup>1</sup>

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The asymptotic behavior of the spectral function of a selfadjoint elliptic operator has been studied extensively; cf. the discussion in [1] and [7]. Recently Agmon and Kannai [2] and Hörmander [8] have obtained error estimates for general operators. Most of this work is concerned with interior estimates for operators with rather smooth coefficients. Here we consider behavior up to the boundary, with minimal assumptions on the coefficients. Details and proofs will appear elsewhere.

Let  $\mathcal{A} = \sum a_\alpha(x) D^\alpha$  be an operator of order  $m = 2r$  defined on a region  $\Omega$  in  $R^n$ . We assume that the boundary  $\partial\Omega$  is uniformly regular of class  $m+1$  in the sense of [6]. Let  $B_j = \sum b_{j,\beta}(x) D^\beta$ ,  $j = 1, 2, \dots, r$ , be an operator of order  $m_j < m$  defined on an  $\epsilon$ -neighborhood of  $\partial\Omega$ . Suppose  $0 < h \leq 1$ . We assume

- (1)  $\mathcal{A}$  is uniformly strongly elliptic on  $\Omega$ .
- (2)<sub>h</sub> The coefficients  $a_\alpha$  are bounded and measurable on  $\Omega$ . For  $|\alpha| = m$  and  $x, y$  in  $\Omega$ ,

$$|a_\alpha(y) - a_\alpha(x)| \leq c |y - x|^h.$$

- (3)<sub>h</sub> The coefficients  $b_{j,\beta}$  and their derivatives of order  $\leq m - m_j$  are bounded and continuous on  $\Omega$ . For  $|\beta| = m_j$  and  $|\gamma| = m - m_j$ ,

$$|D^\gamma b_{j,\beta}(y) - D^\gamma b_{j,\beta}(x)| \leq c |y - x|^h.$$

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