RESTRICTED LIE ALGEBRAS OF BOUNDED TYPE

BY RICHARD D. POLLACK[1]

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Introduction. It is known [13] that a Lie algebra over a modular field has indecomposable representations of arbitrarily high dimensionalities. If, however, the Lie algebra and its representations are required to be restricted (see [6, Chapter 5] for definitions), this need no longer be the case.

A restricted Lie algebra for which the degrees of its (restricted) indecomposable representations are bounded by some constant is said to be of *bounded type*; one for which this is not the case is said to be of *unbounded type.*

1. **The simple three-dimensional Lie algebra,** A_1. Let A_1 be the split simple three-dimensional Lie algebra over the field K of characteristic $p>3$. Then A_1 has a basis e, f, h with $[e,f]=h$, $[e,h]=2e$, $[f,h]=-2f$ and with p-power mapping given by $e^p=f^p=0$, $h^p=h$. There are p inequivalent irreducible (restricted) modules for A_1, classified by their highest weight. Let M_λ, $0\leqq\lambda\leqq p-1$, be the irreducible A_1-module with highest weight λ, so that $[M_\lambda : K]=\lambda+1$ [5].

Let U be the u-algebra [6] of A_1 and $U=\sum_{j=1}^{n}\oplus U_j$ its decomposition into its principal indecomposable modules (p.i.m.). Since U is a symmetric algebra [9] each U_j has a unique top and bottom composition factor, these are isomorphic, and each M_λ is isomorphic to the top composition factor of some U_j [2].

If M is an A_1-module, denote by $M\sim M_{\lambda_1}, M_{\lambda_2}, \cdots, M_{\lambda_s}$ the fact that the M_{λ_j}, in the given order, are the composition factors of some composition series for M.

THEOREM 1. *Let $U(\lambda)$, $0\leqq\lambda\leqq p-1$, be a p.i.m. of U whose top composition factor is isomorphic to M_λ. Then*

(i) *$U(p-1)\cong M_{p-1}$ and $[U(p-1): K]=p$.*

(ii) *If $\lambda\neq p-1$, then $U(\lambda)\sim M_\lambda, M_\gamma, M_\gamma, M_\lambda$, where $\lambda+\gamma=p-2$, and $[U(\lambda):K]=2p$.*

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