GEOMETRIC PROGRAMMING: A UNIFIED DUALITY THEORY FOR QUADRATICALLY CONSTRAINED QUADRATIC PROGRAMS AND l_p-CONSTRAINED l_p-APPROXIMATION PROBLEMS[1]

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The duality theory of geometric programming as developed by Duffin, Peterson, and Zener [1] is based on abstract properties shared by certain classical inequalities, such as Cauchy's arithmetic-geometric mean inequality and Hölder's inequality. Inequalities with these abstract properties have been termed "geometric inequalities" ([1, p. 195]). We have found a new geometric inequality, which we state below, and have used it to extend the "refined duality theory" of geometric programming developed by Duffin and Peterson ([2] and [1, Chapter VI]). This extended duality theory treats both quadratically-constrained quadratic programs and l_p-constrained l_p-approximation problems. By a quadratically constrained quadratic program we mean: to minimize a positive semidefinite quadratic function, subject to inequality constraints expressed in terms of the same type of functions. By an l_p-constrained l_p-approximation problem we mean: to minimize the l_p norm of the difference between a fixed vector and a variable linear combination of other fixed vectors, subject to inequality constraints expressed by means of l_p norms.

Both the classical unsymmetrical duality theorems for linear programming (Gale, Kuhn and Tucker [3], and Dantzig and Orden [4]) and the unsymmetrical duality theorems for linearly-constrained quadratic programs (Dennis [5], Dorn [6], [7], Wolfe [8], Hanson [9], Mangasarian [10], Huard [11], and Cottle [12]) can be derived from the extended duality theorems that we state below and have proved on the basis of the new geometric inequality.

The new geometric inequality is

$$\sum_{1}^{N+1} x_i y_i \leqq y_{N+1}\left(\sum_{1}^{N} p_i^{-1} \left| x_i - b_i \right|^{p_i} + (x_{N+1} - b_{N+1}) \right) + \sum_{1}^{N} \left(q_i^{-1} y_{N+1}^{(1-q_i)} \left| y_i \right|^{q_i} + b_i y_i \right) + b_{N+1} y_{N+1},$$

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