FINITE LINEAR GROUPS IN SEVEN VARIABLES[1]

BY DAVID B. WALES

Communicated by Walter Feit, August 22, 1967

If G is a finite group which has a faithful complex representation of degree n it is said to be a linear group in n variables. This is equivalent to saying G is a finite group of complex linear transformations. It is customary to consider only unimodular linear transformations. For $n \leqq 4$ these groups have been known for a long time. An account may be found in Blichfeldt's book [1]. For $n=5$ they were determined by R. Brauer in [2]. Results in [2] are used to prove the following theorem for $n=7$.

THEOREM 1. *If G has a complex irreducible representation of degree 7 which is faithful, unimodular, and primitive, then G is one of the following groups. Here $Z(G)$ is the center of G.*

I. *G is a uniquely determined group of order $7^4 \cdot 48$ which has a normal subgroup D of order 7^3, $G/D \cong \mathrm{SL}(2, 7)$. D is nonabelian with exponent* 1.

II. *Certain subgroups of G in* I *of order $7^3 \cdot s$ where $s \mid 48$. These contain D.*

III. $G/Z(G) \cong \mathrm{PSL}(2, 13)$	$\lvert G: Z(G) \rvert = 13 \cdot 7 \cdot 3 \cdot 2^2$.
IV. $G/Z(G) \cong \mathrm{PSL}(2, 8)$	$\lvert G: Z(G) \rvert = 7 \cdot 3^2 \cdot 2^3 = 504$.
V. $G/Z(G) \cong A_8$	$\lvert G: Z(G) \rvert = 8!/2$.
VI. $G/Z(G) \cong \mathrm{PSL}(2, 7)$	$\lvert G: Z(G) \rvert = 7 \cdot 3 \cdot 2^3 = 168$.
VII. $G/Z(G) \cong \mathrm{PSU}(3, 9)$	$\lvert G: Z(G) \rvert = 7 \cdot 3^3 \cdot 2^5 = 6048$.
VIII. $G/Z(G) \cong S_6(2)$	$\lvert G: Z(G) \rvert = 7 \cdot 5 \cdot 3^4 \cdot 2^9$.

IX. *$G/Z(G)$ is an extension of* V, VI, VII *by an automorphism of order 2 or an extension of* IV *by an automorphism of order 3. For* V *it is S_8, for* VI *it is induced by* PL(2, 7). *For* VII *it is induced by a field automorphism and is $G_2(2)$. The extension of* IV *is induced by a field automorphism.*

REMARKS. a. In the cases III–IX, $Z(G)$ has order 1 or 7. If it has order 7 there is a subgroup G_1 such that $G \cong G_1 \times Z(G)$.

b. A group satisfying all the hypotheses of Theorem 1 except for primitivity is a monomial group. In this case there is a normal abelian

[1] This work was the author's doctoral dissertation at Harvard University under the supervision of Professor R. Brauer. It was supported by the Canadian National Research Council.