2. ———, *Types of ultrafilters*, Proc. of the 2nd Topology Symposium 1966, Academia 1967, Praha.

3. M. Katětov, *A theorem on mappings*, Comment Math. Univ. Carolinae. **8** (1967), 431–433.

4. R. Raimi, *Homeomorphisms and invariant measures for $\beta N-N$*, Duke Math. J. **13** (1966), 1–12.

5. W. Rudin, *Homogeneity problems in the theory of Čech compactification*, Duke Math. J. **23** (1956), 409–420.

LOUISIANA STATE UNIVERSITY, BATON ROUGE,
UNIVERSITY OF CALIFORNIA, RIVERSIDE, AND
MATHEMATICS INSTITUTE, PRAHA

REPRESENTATION OF FRACTIONAL POWERS OF INFINITESIMAL GENERATORS OF SEMIGROUPS

BY H. BERENS, P. L. BUTZER AND U. WESTPHAL[1]

Communicated by Felix Browder, September 1, 1967

1. Introduction. Let X be a real or complex Banach space and $\mathfrak{E}(X)$ the Banach algebra of endomorphisms of X. Let $\{T(u); u \geqq 0\}$ be an equibounded semigroup of operators of class $(\mathfrak{C}_0)$ in $\mathfrak{E}(X)$ with infinitesimal generator A. A is a closed linear operator with domain $D(A)$ dense in X. For these concepts see e.g. E. Hille-R. S. Phillips [5, Chapters X–XII].

One purpose of this note is to give a representation for the fractional power $(-A)^\gamma$, $\gamma > 0$, of the operator $(-A)$. The result obtained will be a generalization of one due to J. L. Lions-J. Peetre [7, Chapter VII, §2]: *Let γ be a positive integer. An element $f \in X$ belongs to $D((-A)^\gamma)$ if and only if the integral*

$$\frac{1}{C_{\gamma,r}} \int_\epsilon^\infty \frac{[I - T(u)]^r f}{u^{1+\gamma}}\, du \qquad (\epsilon > 0;\ r \text{ any integer } > \gamma). \tag{1}$$

converges in norm as $\epsilon \to 0+$, the constant $C_{\gamma,r}$ being given by

$$C_{\gamma,r} = \int_0^\infty \frac{(1 - e^{-u})^r}{u^{1+\gamma}}\, du. \tag{2}$$

The limit is then equal to $(-A)^\gamma f$.

[1] The research of this author was supported by a DFG grant.