REPRESENTATIONS OF LOCALLY FINITE GROUPS

BY DAVID J. WINTER[1]

Communicated by G. B. Seligman, August 15, 1967

The purpose of this paper is to give a brief general account of the completely reducible finite-dimensional representations of a locally finite group G over a given algebraically closed field K. Theorem 1 shows that all such representations of G can be brought down to the algebraic closure F in K of the prime field of K. This reduces all further considerations in this account to countable groups. Theorem 2 characterizes the existence of a faithful completely reducible representation of G of degree n over K in terms of the existence of such representations for appropriate finite subgroups of G.

Throughout the paper, G denotes a locally finite group, K denotes an arbitrary algebraically closed field and F denotes the algebraic closure in K of the prime field of K. V denotes an n-dimensional vector space over K. An *F-form of V* is an F-subspace W of V such that W and K are linearly disjoint over K and V is the K-span of W. (Equivalently, an F-form of V is the F-span of a basis of V.) If A is an F-algebra, A_K denotes the algebra $A \otimes_F K$.

THEOREM 1. *Let ρ be a completely reducible representation of G in V. Then V has an F-form W which is stable under the ρ-action of G in V.*

PROOF. It suffices by complete reducibility to consider the case in which G acts irreducibly in V.

If G is finite, the assertion follows (upon passing to the group algebra of G over F) from the fact that if A is a finite-dimensional associative algebra over F, then an irreducible (finite-dimensional) A_K-module has an F-form stable under A. Since the kernel of an irreducible representation of such an A contains the radical of A, it suffices to prove this in the case that A is semisimple. But for A semisimple, the assertion is obvious since:

(1) $A = \sum_1^m \oplus A_i$ where $A_1, \cdots, A_m$ are minimal right ideals of A;

(2) $A_K = \sum_1^m \oplus (A_i)_K$ and the $(A_i)_K$ are minimal right ideals of A_K;

(3) any irreducible A_K-module is isomorphic to one of the $(A_i)_K$ [1, Chapter IV].

Next assume that G is locally finite and that (ρ, V) is an irreducible representation of G over K of degree n. Then some finite subgroup H of G acts irreducibly in V. (For example let S be a finite subset of G

[1] This research was partially supported by research grant NSF-GP-4017.