THE POISSON-LAGUERRE TRANSFORM

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For $\alpha \geqq 0$, let $L_n^\alpha(x)$ denote the Laguerre polynomial of degree n given by

$$L_n^\alpha(x) = (x^{-\alpha} e^x / n!)(d/dx)^n (x^{n+\alpha} e^{-x}), \qquad n = 0, 1, \cdots.$$

We define the Laguerre difference operator ∇_n by

$$\nabla_n f(n) = (n+1) f(n+1) - (2n + \alpha + 1) f(n) + (n + \alpha) f(n-1).$$

Then the Laguerre difference heat equation is given by

$$(*) \qquad \nabla_n u(n, t) = \partial u(n, t)/\partial t.$$

A Laguerre temperature is a solution $u(n, t)$ of (*) which is a C^1 function of t. The fundamental Laguerre temperature is the function $g(n; t) = g(n, 0; t)$, where

$$g(n, m; t) = \int_0^\infty e^{-xt} L_n^\alpha(x) L_m^\alpha(x) d\Omega(x), \qquad t > 0,$$

with

$$d\Omega(x) = e^{-x} x^\alpha dx.$$

Corresponding to $g(n, m; t)$ is its conjugate $g(n^*, m; t)$ given by

$$g(n^*, m; t) = \int_0^\infty e^{-xt} L_n^\alpha(-x) L_m^\alpha(x) d\Omega(x), \qquad t > 0.$$

An important subclass of the class of Laguerre temperatures includes those Laguerre temperatures $u(n, t)$ which satisfy the condition

$$u(n, t) = \sum_{m=0}^\infty g(n, m; t - t') u(m, t') \rho(m), \qquad \rho(m) = m!/\Gamma(m + \alpha + 1),$$

for every t, t', $0 < t' < t$, with the series converging absolutely. Laguerre temperatures which belong to this subclass are said to have the Huygens property. The functions $g(n, m; t)$ have this property.

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