GLOBAL RATIO LIMIT THEOREMS FOR SOME NONLINEAR FUNCTIONAL-DIFFERENTIAL EQUATIONS. I

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1. **Introduction.** We study some systems of nonlinear functional-differential equations of the form

$$\dot{X}(t) = AX(t) + B(X_t)X(t-\tau) + C(t), \quad t \geqq 0, \tag{1}$$

where $X=(x_1, \cdots, x_n)$ is nonnegative, $B(X_t)=\|B_{ij}(t)\|$ is a matrix of nonlinear functionals of $X(w)$ evaluated at all past times $w\in[-\tau, t]$, and $C=(C_1, \cdots, C_n)$ is a known nonnegative and continuous input function. For appropriate A, B, and C, these systems can be interpreted as a nonstationary prediction theory whose goal is to discuss the prediction of individual events, in a fixed order, and at prescribed times, or alternatively as a mathematical learning theory. This interpretation is discussed in a special case in [1]. The systems can also be interpreted as cross-correlated flows on networks, or as deformations of probabilistic graphs.

The mathematical content of these interpretations is contained in assertions of the following kind: given arbitrary positive and continuous initial data along with a suitable input C, the ratios $y_{jk}(t)=B_{kj}(t)(\sum_{m=1}^n B_{km}(t))^{-1}$ have limits as $t\to\infty$.

Our systems are defined in the following way. Given any positive integer n; any real numbers α, u, $\beta>0$, and $\tau\geqq 0$; and any $n\times n$ semistochastic matrix $P=\|p_{ij}\|$ (i.e., $p_{ij}\geqq 0$ and $\sum_{m=1}^n p_{im}=0$ or 1), let

$$\dot{x}_i(t) = -\alpha x_i(t) + \beta\sum_{k=1}^n x_k(t-\tau)y_{ki}(t) + C_i(t), \tag{2}$$

$$y_{jk}(t) = p_{jk}z_{jk}(t)\left[\sum_{m=1}^n p_{jm}z_{jm}(t)\right]^{-1}, \tag{3}$$

and

$$\dot{z}_{jk}(t) = [-uz_{jk}(t) + \beta x_j(t-\tau)x_k(t)]\theta(p_{jk}), \tag{4}$$

for all $i, j, k=1, 2, \cdots, n$, where

$$\theta(p) = 1 \quad \text{if } p>0,$$
$$\qquad = 0 \quad \text{if } p\leqq 0.$$

In order that our theorems hold, the initial data must always be non-