STABLE HOMOTOPY CATEGORIES[1,2]

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Introduction. The Freudenthal suspension theorem implies that the set of homotopy classes of continuous maps from one finite complex to another is eventually invariant under iterated suspension of the complexes. In this "stable range" the set of homotopy classes is also well behaved in other ways. For example, it has in a natural way the structure of an abelian group. These stable groups of homotopy classes form the subject matter of stable homotopy theory. The most convenient way to describe them is as the colimit under successive suspensions of the set of homotopy classes. This construction, introduced by Spanier and Whitehead [**19**], gives interesting objects even for pairs of spaces for which the hypotheses of the Freudenthal theorem fail.

In this way the stable homotopy category—or rather categories, since the class of spaces considered may be chosen in many ways or even enlarged to encompass objects, such as spectra, more general than spaces—is defined. Stable homotopy categories have been extensively studied, being the natural loci of such phenomena as the S-duality of Spanier and Whitehead (loc. cit.) and the spectral sequences of Adams [**1**].

In these circumstances Puppe [**17**] was led to propose an intrinsic definition of a stable homotopy category as a category with certain additional structure (cf. §9 below). Stable homotopy categories of spaces are examples of these objects, but others may be derived from algebraic sources, viz. from certain categories of chain complexes. The notion has proved useful and has been exploited both in topological and in algebraic contexts (Heller [**14**], Verdier [**21**], Boardman [**2**], etc).

Freyd is responsible [**9**] for the observation that Puppe's stable homotopy categories are canonically imbedded in abelian categories or, more precisely, that categories which admit the additional structure described by Puppe are so imbedded. The well-developed methods of homological algebra can thus be directly applied to the study of stable homotopy; Freyd has begun the exploitation of these methods [**10**], [**11**].

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