# GENERATORS FOR SOME RINGS OF ANALYTIC FUNCTIONS

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Let $\Omega$ be an open set in $\mathbf{C}^n$ and let $p$ be a nonnegative function defined in $\Omega$. We shall denote by $A_p(\Omega)$ the set of all analytic functions $f$ in $\Omega$ such that for some constants $C_1$ and $C_2$

$$(1) \qquad |f(z)| \leqq C_1 \exp(C_2 p(z)), \qquad z \in \Omega.$$

It is obvious that $A_p(\Omega)$ is a ring. We wish to determine when it is generated by a given finite set of elements $f_1, \cdots, f_N$. There is an obvious necessary condition, for if $f_1, \cdots, f_N$ are generators for $A_p(\Omega)$ we can in particular find $g_1, \cdots, g_N \in A_p(\Omega)$ so that $1 = \sum f_j g_j$. Hence we have

$$1 \leqq \sum |f_j(z)| C_1 \exp(C_2 p(z))$$

for some constants $C_1$ and $C_2$, that is,

$$(2) \qquad |f_1(z)| + \cdots + |f_N(z)| \geqq c_1 \exp(-c_2 p(z)), \qquad z \in \Omega,$$

for some positive constants $c_1$ and $c_2$.

This note concerns the converse statement. Carleson [1] has proved a deep result of that type, called the Corona Theorem, which states that (2) implies that $f_1, \cdots, f_N$ generate $A_p(\Omega)$ if $p=0$ and $\Omega$ is the unit disc in $\mathbf{C}$. In a recent research announcement [5] in this Bulletin, the Corona Theorem was used to prove the analogous result when $p(z) = |z|$ and $\Omega = \mathbf{C}$. However, we shall see here that this statement is much more elementary than the Corona Theorem; indeed, we shall prove a general result of this kind for functions of several complex variables although no analogue of the Corona Theorem is known there.

THEOREM 1. *Let $p$ be a plurisubharmonic function in the open set $\Omega \subset \mathbf{C}^n$ such that*

(i) *all polynomials belong to $A_p(\Omega)$;*

(ii) *there exist constants $K_1, \cdots, K_4$ such that $z \in \Omega$ and $|z - \zeta| \leqq \exp(-K_1 p(z) - K_2) \Rightarrow \zeta \in \Omega$ and $p(\zeta) \leqq K_3 p(z) + K_4$.*

*Then $f_1, \cdots, f_N \in A_p(\Omega)$ generate $A_p(\Omega)$ if and only if (2) is valid.*

Before the proof we make a few remarks. First note that if $d(z)$