NONLINEAR EQUATIONS OF EVOLUTION AND NONLINEAR ACCRETIVE OPERATORS IN BANACH SPACES

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Introduction. Let X be a real Banach space, T a (possibly) nonlinear mapping with domain $D(T)$ in X and range $R(T)$ in X. Following [7] and [8], we shall say that T is *accretive* if for all x and y in $D(T)$,

$$(T(x) - T(y), J(x - y)) \geqq 0, \tag{1}$$

where (assuming that the conjugate space X^* of X is strictly convex), J is the mapping of X into X^* which assigns to each x of X the bounded linear functional $w = J(x)$ such that $(w, x) = \|w\| \cdot \|x\|$ and $\|w\| = \|x\|$.

It is our object in the present note to present some new and sharper results on two related topics:

(1) The existence theory of solutions for the initial value problem for nonlinear equations of evolution of the form

$$du/dt + T(t)u(t) = f(t, u(t)) \quad (t \geqq 0) \tag{2}$$

with the initial condition $u(0) = x_0$.

(2) The existence theory of solutions of the equation

$$T(u) = w, \tag{3}$$

for an accretive operator T and an element w of X.

In the study of the equation of evolution (2), we assume that for each t in R^+, $T(t)$ is an accretive operator such that $D(T(t))$ is independent of t and $R(T(t)+I) = X$, while f is a continuous, bounded mapping of $R^+ \times X$ into X. For the special case that X is a Hilbert space and $T(t)$ is linear, such results were obtained in Browder [1] and Kato [12], and extensions for $T(t)$ linear and more general Banach spaces X were given in Murakami [17] and Browder [7] (and in an earlier version of [8]). Results for $T(t)$ nonlinear were first obtained by Komura [15] in Hilbert space and extended to more general Banach spaces by Kato [13] for the case in which $f = 0$. Our proofs (which are given in detail in [8]) and those of Kato [13] are based upon an elementary device applied by Murakami in [17].

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