RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited. Manuscripts more than eight typewritten double spaced pages long will not be considered as acceptable.

A CONNECTION BETWEEN α-CAPACITY AND $m-p$ POLARITY

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In investigating questions of removable singularities of second order partial differential equations, Serrin [3] introduced the notion of the M_s capacity of a compact set S in R^n as follows:

$$M_s(S) = \inf \int |\operatorname{grad} u|^s \, dx$$

where the infimum is taken over all continuously differentiable functions u having compact support in R^n and $\geqq 1$ on S. It turns out that in this definition the "$\geqq$" sign may be replaced by an equal sign. Indeed it is the definition using the equal sign which is really made use of in the proofs. The equivalence of the two definitions is, roughly speaking, due to the fact that, in taking the infimum, each competing function u may be truncated, i.e., be replaced by $\bar{u} \equiv \min (u, 1)$. $\bar{u}$ will not in general be continuously differentiable, but this difficulty can be overcome.

There is a more classical notion of capacity, $C_\alpha(S)$ due to Frostman and others. (See [4], for example, for a brief description of the relevant properties of C_α.) Wallin [4] has exhibited a close relationship between $M_s(S) = 0$ and $C_\alpha(S) = 0$ for appropriately related values of α and s.

In order to investigate certain questions of removable singularities for higher order partial differential equations, the author [1] has found that the appropriate concept for the smallness of a set was that of $m-p$ polarity. We define, for a compact set S in R^n,

$$M_{m,p}(S) \equiv \inf |u|^p_{m,p} \qquad (1)$$

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