NONSTANDARD ARITHMETIC[1]

ABRAHAM ROBINSON

1. Introduction. In 1934 it was pointed out by Thoralf Skolem [23] that there exist proper extensions of the natural number system which have, in some sense, "the same properties" as the natural numbers. As the title of his paper indicates, Skolem was interested only in showing that no axiomatic system specified in a formal language (in his case the Lower Predicate Calculus) can characterize the natural numbers categorically; and he did not concern himself further with the properties of the structures whose existence he had established. In due course these and similar structures became known as *nonstandard models of arithmetic* and papers concerned with them, wholly or in part, including certain applications to other fields, appeared in the literature (e.g. [7], [9], [11], [14], [15], [16], [17]). Beginning in the fall of 1960, the application of similar ideas to analysis led to a rapid development in which nonstandard models of arithmetic played an auxiliary but vital part. It turned out that these ideas provide a firm foundation for the nonarchimedean approach to the Differential and Integral Calculus which predominated until the middle of the nineteenth century when it was discarded as unsound and replaced by the ϵ, δ method of Weierstrass. Going beyond this area, which is particularly interesting from a historical point of view, the new method (which has come to be known as *Nonstandard Analysis*) can be presented in a form which is sufficiently general to make it applicable also to mathematical theories which do not involve any metric concept, e.g., to general topological spaces [18].

In the present paper we shall show how the experience gained with this more general approach can be used in order to throw new light also on arithmetic or more precisely, on the classical arithmetical theories which have grown out of elementary arithmetic, such as the theory of ideals, the theory of p-adic numbers, and class field theory. Thus we shall provide new foundations for infinite Galois theory and for the theory of idèles. Beyond that, we shall develop a theory of *ideals* for the case of infinite abelian extensions in class field theory. This is remarkable, for Chevalley introduced idèles [2] precisely in

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