INVOLUTIONS OF HOMOTOPY SPHERES AND HOMOLOGY 3-SPHERES

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1. **Introduction.** Let Σ^{4n+3} be a homotopy sphere and $T\colon \Sigma\to\Sigma$ a fixed point free differentiable involution. A characteristic submanifold for T is a smoothly embedded submanifold $W^{4n+2}\subset\Sigma$, such that $W=A\cap TA$, $\Sigma=A\cup TA$, where A is a compact submanifold of Σ, $\partial A=W$. Let $i\colon W\to A$ be the inclusion, and $K=\ker i_*$, $i_*\colon H_{2n+1}(W)\to H_{2n+1}(A)$. The symmetric bilinear pairing $K\otimes K\to \boldsymbol{Z}$ defined by $x\otimes y\to x\cdot T_*y$ is called the quadratic form of T with respect to W, and its signature is denoted by $\sigma(T,\Sigma)$. It is proved in [2] that $\sigma(T,\Sigma)$ does not depend on the characteristic submanifold, and that for $n>0$, $\sigma(T,\Sigma)=0$ if and only if Σ contains an invariant smoothly embedded S^{4n+2}. These definitions can be made in the p.l. category and the corresponding properties hold. $\sigma(T,\Sigma)$ can also be defined when Σ is a homology sphere.

It is the purpose of this paper to give examples of involutions with $\sigma(T,\Sigma)\neq 0$.

2. We will make use of the following construction: Let M^n be a smooth manifold, and $T\colon\partial M\to\partial M$ a smooth involution. Consider another copy M^* of M, and the manifold $M'=M\cup_T M^*$, obtained from the disjoint union of M and M^* by identifying $T(x)\in\partial M$ with $x^*\in\partial M^*$. Then an involution $T'\colon M'\to M'$ can be defined by $T'(x)=x^*$, $T'(x^*)=x$. $T'|\partial M=T$ and T' is fixed point free if and only if T is.

We will denote by U the square matrix with 1's in the nonprincipal diagonal and 0's elsewhere.

Let H be a $2k\times 2k$ integral matrix. We will consider the following conditions on H:

(i) $\det H=\pm 1$.

(ii) There exist $2k\times 2k$ integral matrices P, Q, such that $H=PUP^t-QUQ^t$.

(iii) PQ^t is symmetric.

(iv) PQ^t has even integers in the main diagonal.

3. THEOREM 1. *If H satisfies conditions* (i)–(iv), *then H can be realized as the matrix of the quadratic form of a fixed point free differentiable involution of a homotopy* $(4n+3)$-*sphere*, $n>0$.