QUANTIZATION AND REPRESENTATIONS OF SOLVABLE LIE GROUPS

BY L. AUSLANDER AND B. KOSTANT[1]

Communicated by Felix Browder, April 10, 1967

Introduction. In this note, we will announce a characterization of a connected, simply connected Type I solvable Lie group, G, and present a complete description of the set of all unitary equivalence classes of irreducible unitary representations of G together with a construction of an irreducible representation in each equivalence class. This result subsumes the results previously obtained on nilpotent Lie groups and solvable Lie groups of exponential type of Kirillov [3] and Bernat [2], respectively.

Our result is made possible by a merging of a new general geometric approach to representation theory, based on the use of symplectic manifolds and quantization, of the second author with a detailed analysis of the Mackey inductive procedure which augments the results in [1].

1. **Outline of results.** Let (X, ω) be a symplectic manifold; i.e., a $2n$-dimensional manifold with a closed 2-form ω such that ω^n does not vanish on X and $d\omega=0$ on X. Let $[\omega]\in H^2(X, R)$ be the corresponding deRham class. A vital example of a symplectic manifold, for our purposes, is obtained as follows: Let G be a Lie group with Lie algebra $\mathfrak{g}$ and let $\mathfrak{g}'$ be the dual vector space to $\mathfrak{g}$. Then G acts on $\mathfrak{g}'$ by the contragredient representation and we will denote a G-orbit by O and the set of G-orbits by $\mathfrak{O}$. After several identifications it is possible to use the bilinear form $\langle f, [x, y]\rangle$, $x, y\in\mathfrak{g}, f\in\mathfrak{g}'$ to define a 2-form ω_O on each O such that (O, ω_O) is a symplectic manifold.

THEOREM 1. *Let G be a connected, simply connected solvable Lie group. Then G is Type* I *if and only if*

(a) *all G-orbits in $\mathfrak{g}'$ are G_δ sets in the usual topology on $\mathfrak{g}$.*

(b) $[\omega_O]=0$ *for all* $O\in\mathfrak{O}$.

REMARK. All algebraic Lie groups are Type I.

In general if (X, ω) is any symplectic manifold then there exists a complex line bundle L with connection α such that ω is the curvature form of the connection α, $\omega=\text{curv}(L, \alpha)$, if and only if the deRham

[1] The first author received partial support from the N.S.F. and U. S. Naval Research Laboratory and the second author received partial support from the N.S.F. and the Sloan Foundation.