THE WEAKLY COMPLEX BORDISM OF LIE GROUPS

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1. **Preliminaries.** Let $\mathcal{K}$ be the class of compact 1 connected semisimple Lie groups; $\mathcal{K}' \subset \mathcal{K}$ is the following set of groups, $\mathrm{Sp}(n)$, $\mathrm{SU}(n)$, $\mathrm{Spin}(n)$, G_2, F_4, E_6, E_7, E_8, $U_*(X)$ the weakly complex bordism of X [1] and Λ the ring $U_*(pt) = Z[Y_1, Y_2, \cdots]$. Λ is the weakly complex bordism ring defined by Milnor. The generators Y_i are weakly complex manifolds of dim $2i$. The bordism class of a weakly complex manifold M^{2n} is determined by its Milnor numbers [2] $s_\omega[M^{2n}]$ for ω ranging over all partitions of n. In particular, the generators Y_i can be chosen so that $s_i(Y_i) = 1$ unless $i = p^k - 1$ for some prime p and in this case $s_i(Y_i) = p$; moreover, we assume generators Y_i chosen so that its Todd genera are 1.

It is possible and convenient to introduce bordism theories with other coefficient rings than Λ. If Γ is such a ring, $U_*(\ ,\Gamma)$ will denote the resulting theory. Briefly here are some examples: $\Lambda_p = Z_p[Y_1, Y_2, \cdots]$, $\Lambda[1/Y_{p-1}] = \text{direct lim } 1/Y_{p-1}^n\Lambda$ and $\Lambda_p[1/Y_{p-1}] = \text{direct lim } 1/Y_{p-1}^n\Lambda_p$.[1] Let $M = \{M_n\}$ denote the stable object of Milnor [1] and $Z_p = S^1 U_p E^2$ the space obtained by attaching E^2 to S^1 via a map of degree p. $M_{n+2}^{Z_p}$ denotes the space of base point preserving maps from Z_p to M_{n+2}. Then $U_k(X, \Lambda_p) = \text{direct lim } \Pi_{n+k}(X^+ \wedge M_{n+2}^{Z_p})$ X^+ is the disjoint union of X and a point $x_0 \cdot U_*(X, \Lambda_p)$ is the resulting theory. $U_*(X, \Lambda[1/Y_{p-1}]) = U_*(X) \otimes_\Lambda \Lambda[1/Y_{p-1}]$ and $U_*(X, \Lambda_p[1/Y_{p-1}]) = U_*(X, \Lambda_p) \otimes_{\Lambda_p} \Lambda_p[1/Y_{p-1}]$.

To $K \subset \mathcal{K}$ there is associated a "generating variety" K_s introduced by Bott [4]. Essentially K_s is the homogeneous space K/K^s where K^s is the centralizer of a 1-dimensional torus $S^1 \subset K$. The dimension of the center of K^s is 1. The commutator map

$$S^1 \times K_s \xrightarrow{[\ ]} K$$

defined by $[t, [k]] = tkt^{-1}k^{-1}$ for $[k] \in K_s$, $t \in S^1 \subset K$ is of particular importance.

2. **Statement of results.** Define $\Lambda(K) = \Lambda$ if $H^*(K)$ has no torsion, $= \Lambda[1/Y_1]$ if $H^*(K)$ has only 2 torsion, $= \Lambda[1/Y_1, 1/Y_2]$ if $H^*(K)$ has only 2, 3 torsion, $= \Lambda[1/Y_1, 1/Y_2, 1/Y_4]$ if $H^*(K)$ has 2, 3 and 5 torsion.

[1] E.g., $\Lambda[1/Y_{p-1}]$ is the ring obtained from Λ by making Y_{p-1} a unit.