THE WEIERSTRASS TRANSFORMATION OF CERTAIN GENERALIZED FUNCTIONS[1]

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The convolution transformation

$$F(s) = \int_{-\infty}^{\infty} f(t)G(s-t)dt \tag{1}$$

considered by Hirschman and Widder [1] has a kernel G of the form

$$G(\tau) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} \frac{\exp(z\tau)}{E(z)}dz \tag{2}$$

where

$$E(z) = \exp(-cz^2 + bz)\prod_{\nu=1}^{\infty}\left(1 - \frac{z}{a_\nu}\right)\exp(z/a_\nu), \tag{3}$$

the c, b, and a_ν are real, $c \geqq 0$, $a_\nu \neq 0$, $|a_\nu| \to \infty$, and $\sum a_\nu^{-2} < \infty$. In a previous note [2] we extended the convolution transformation to a certain class of generalized functions in the case where $c=0$ in (3). On the other hand, if we substitute the previously neglected factor $\exp(-cz^2)$ in place of $E(z)$ in (2) and normalized by setting $c=1$, we obtain

$$G(\tau) = k(\tau, 1) \tag{4}$$

where

$$k(\tau, t) = \exp(-\tau^2/4t)/(4\pi t)^{1/2}, \quad -\infty < \tau < \infty, \quad 0 < t < \infty.$$

The convolution transformation (1) then becomes the Weierstrass transformation [1; Chapter VIII]:

$$F(s) = \frac{1}{(4\pi)^{1/2}}\int_{-\infty}^{\infty} f(\tau)\exp[-(s-\tau)^2/4]d\tau. \tag{5}$$

Here, we round out our previous results by extending (5) to certain generalized functions.

Let a and b be fixed real numbers with $a < b$. Let $\rho_{a,b}(\tau)$ be a posi-

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