AUTOMORPHISMS OF THICKENINGS

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An m-thickening of a CW-complex K^k (see [7]) consists of a simple homotopy equivalence $\psi: K^k_{1*} \to M_{1*}$, where $m \geqq k+3$, together with an orientation of the tangent space at the base point. In this note we associate to any PL (or smooth) manifold, semisimplicial complexes corresponding to the homotopy equivalences of the manifold with itself, and to PL (or smooth) automorphisms. By giving a suitable definition of a relative complex we obtain a long exact sequence of homotopy groups. In the case where we restrict our manifolds to be thickenings of certain 'good' complexes we are able to give an interpretation of the relative terms, by means of the structure of certain thickenings. As a consequence we are able to describe the group of quasi-isotopy classes of PL-automorphisms of $S^p \times S^q$ which are homotopic to the identity. The full details will appear elsewhere. This note may be regarded as bearing much the same relation to [7] as [6] does to [5].

As in [7] the bulk of the material is applicable to both the piecewise linear and smooth categories. The terms homeomorphism, manifold, etc., should therefore be interpreted accordingly.

Definitions and notation. Let M^m be a compact connected manifold, and $D^m_* \subset M^m$ a disc containing the base point of M, in case M has boundary we suppose $\partial M \cap D_* = D^{m-1}_* \ni *$. We now define the semisimplicial complexes with which we shall be working.

The complex $\mathcal{E}(M)$. A k-simplex of $\mathcal{E}(M)$ is a homotopy equivalence

$$F: \Delta^k \times M \to \Delta^k \times M$$

such that

(1) If Δ^s is a face of Δ^k the inclusion induces $F': \Delta^s \times M \to \Delta^s \times M$.

(2) $F | \Delta^k \times D^m_* = \text{Identity}$.

The complex $\mathfrak{D}(M)$. Here it is necessary to make a distinction between the piecewise linear and smooth cases.

Smooth case. A k-simplex of $\mathfrak{D}(M)$ is a map

$$F: \Delta^k \times M \to \Delta^k \times M$$

such that