A THEOREM ON RANK WITH APPLICATIONS TO MAPPINGS ON SYMMETRY CLASSES OF TENSORS

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Communicated by Gian-Carlo Rota, March 31, 1967

1. **Results.** Let R be a field containing a real closed subfield R_0. The main results of this announcement follow.

THEOREM 1. *Let $A_1, A_2, \cdots, A_p$ be $m \times n$ matrices with entries in an infinite subset Ω of R containing the natural numbers in R_0. Let k be a positive integer and assume that the rank of each A_i is at least k. Then there exist nonsingular matrices E and F with entries in Ω such that every set of k rows (columns) of EA_iF is linearly independent, $i=1, \cdots, p$.*

COROLLARY 1. *If the matrices $A_1, \cdots, A_p$ in Theorem 1 each have rank precisely k then every k-square subdeterminant of EA_iF is nonzero, $i=1, \cdots, p$.*

THEOREM 2. *If $A_1, \cdots, A_p$ are n-square complex hermitian matrices all of rank at least k then there exists a nonsingular matrix E such that every set of k rows (columns) of E^*A_iE is linearly independent.*

In 1933, J. Williamson [1] gave necessary and sufficient conditions for the compounds of two matrices to be equal. The nontrivial part of his result states the following: *if A is a complex matrix of rank r and $r > m$ then $C_m(A) = C_m(B)$ if and only if $A = zB$ where $z^m = 1$.* A result closely connected to Theorem 1 and generalizing the Williamson result can be proved. We state our theorem in an invariant setting.

Thus, let V be an n-dimensional space over the complex numbers, let H be a subgroup of the symmetric group S_m, $m \leqq n$, and let χ be a complex valued character of degree 1 on H. A multilinear function $f(v_1, \cdots, v_m)$ is symmetric with respect to H and χ if $f(v_{\sigma(1)}, \cdots, v_{\sigma(m)}) = \chi(\sigma)f(v_1, \cdots, v_m)$ for all $v_1, \cdots, v_m$ in V and all $\sigma \in H$. Let P be a vector space and f a fixed multilinear function symmetric with respect to H and χ, f: $V \times \cdots \times V \rightarrow P$, such that for any multilinear function g, g: $V \times \cdots \times V \rightarrow U$, also symmetric with respect to H and χ, there exists a linear h: $P \rightarrow U$ that makes the following diagram commutative:

[1] This research was completed under Grant AFOSR 698-67 awarded by the Air Force Office of Scientific Research.