GENERALIZATION OF SCHWARZ-PICK LEMMA TO INVARIANT VOLUME IN A KÄHLER MANIFOLD

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Let $\mathfrak{D}$ be the class of bounded homogeneous domains D in the space C^n of n complex variables $z=(z^1, \cdots, z^n)$. A domain D is *homogeneous* if any point of D can be mapped into any other by a holomorphic automorphism. A bounded domain D possesses the Bergman metric, which is invariant under biholomorphic mappings of D, given by

$$ds_D^2 = T_{\alpha\bar{\beta}} dz^\alpha d\bar{z}^\beta \tag{1}$$

(the summation convention is used), where

$$\begin{aligned} T_{\alpha\bar{\beta}} &= T_{\alpha\bar{\beta}}(z, \bar{z}) = (\partial^2 \log K_D)/(\partial z^\alpha \partial \bar{z}^\beta), \\ T_D &= T_D(z, \bar{z}) = \det(T_{\alpha\bar{\beta}}), \end{aligned} \tag{2}$$

and $K_D = K_D(z, \bar{z})$ is the Bergman kernel function of D [2]. The functions $K_D(z, \bar{z})$ and $T_D(z, \bar{z})$ are *relative invariants* of D under biholomorphic mappings and consequently the function

$$I_D(z, \bar{z}) = K_D(z, \bar{z})/T_D(z, \bar{z}) \tag{3}$$

is an *invariant* of D. The kernel function $K_D(z, \bar{z})$ becomes infinite on the boundary of D.

Let $\mathcal{K}$ be the class of Kähler manifolds Δ with metric given by

$$d\sigma_\Delta^2 = g_{\alpha\bar{\beta}}(w, \bar{w}) dw^\alpha d\bar{w}^\beta, \qquad g_\Delta = g_\Delta(w, \bar{w}) = \det(g_{\alpha\bar{\beta}}), \tag{4}$$

where w is a local coordinate of a point on Δ. We also assume

$$-r_{\alpha\bar{\beta}} u^\alpha \bar{u}^\beta \geqq 0 \qquad \text{for any vector } u = (u^\alpha), \tag{5a}$$

$$\det(-r_{\alpha\bar{\beta}}) \geqq g_\Delta, \tag{5b}$$

where $r_{\alpha\bar{\beta}} = -(\partial^2 \log g_\Delta)/(\partial w^\alpha \partial \bar{w}^\beta)$ are the components of the Ricci curvature tensor of the metric (4).

A domain D is *star-like with respect to a point* $z_0 \in D$ if $z \in D$ implies $r(z-z_0) \in D$ for $0 < r \leqq 1$. If D is star-like, then the image domains D_r of D under the similarity map

$$w = r(z - z_0), \qquad 0 < r \leqq 1 \tag{6}$$