GLOBAL SOLUTIONS OF CERTAIN HYPERBOLIC SYSTEMS OF QUASI-LINEAR EQUATIONS

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Communicated by L. Cesari, March 23, 1967

We consider systems of the form

$$u_t + f(v)_x = 0, \qquad v_t + g(u)_x = 0, \tag{1}$$

with initial data $(v(0, x), u(0, x)) = (v_0(x), u_0(x))$. Here u and v are functions of t and x, $t \geqq 0$, $-\infty < x < \infty$, and f and g are C^2 functions of a single real variable. We assume that the system (1) is hyperbolic and genuinely nonlinear in the sense of Lax [4].

THEOREM 1. *For each point (v_0, u_0) in the $(v-u)$-plane, there exist two smooth curves $u = w(v) = w(v, v_0, u_0)$ and $u = s(v) = s(v, v_0, u_0)$, passing through (v_0, u_0) defined for all $v \geqq v_0$ with the properties that $w'(v) > 0$, $s'(v) < 0$ and each point $(v, w(v))$ satisfies the Lax conditions for backward rarefaction waves [4], while each point $(v, s(v))$ satisfies the Lax conditions for forward shock waves [4].*

In other words, the Riemann problem for (1) with initial data

$$\begin{aligned} (v_0(x), u_0(x)) &= (v_0, u_0), \qquad & x < 0, \\ &= (v_1, w(v_1)), & x > 0 \end{aligned}$$

where $v_1 > v_0$, can be solved by two constant states (v_0, u_0) and $(v_1, w(v_1))$ separated by a backward rarefaction wave. Similarly the Riemann problem for (1) with initial data

$$\begin{aligned} (v_0(x), u_0(x)) &= (v_0, u_0), \qquad & x < 0, \\ &= (v_1, s(v_1)), & x > 0 \end{aligned}$$

where $v_1 > v_0$ can be solved by two constant states (v_0, u_0) and $(v_1, s(v_1))$ separated by a forward shock wave.

Fix a point (v_0, u_0) in $(v-u)$-space and let

$$C(v_0, u_0) = \{(v, u): v \geqq v_0, \quad s(v, v_0, u_0) \leqq u \leqq w(v, v_0, u_0)\}$$

THEOREM 2. *If $(v_1, u_1) \in C(v_0, u_0)$, then $C(v_1, u_1) \subset C(v_0, u_0)$.*

One consequence of Theorem 2 is that the interaction of two forward shocks produces a forward shock and a backward rarefaction

[1] NSF Cooperative Fellow.

[2] Research supported in part by NSF Research Grant GP 3466.