THE SOLUTION SPACES FOR INTEGRAL EQUATIONS OF THE SCATTERING THEORY[1]

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The integral equations expressing the scalar wave functions for the exterior region of a smooth and bounded scatterer in terms of the potential Green's function have recently been found [2], [3]. In the following note we discuss the solutions for these equations.

1. **The statement of the problem.** Let B denote the boundary of a smooth, closed and bounded surface in E^3, and V the exterior of B. Erect a spherical polar coordinate system with origin interior to B; and denote by $\mathbf{r}$ a point (r, θ, ϕ) in V and by $\mathbf{r}_B$ a point (r_B, θ_B, ϕ_B) on B. Further let v be a scalar wave function for the exterior V, i.e.,

(a) $v(\mathbf{r})$ is of class C^2, $\mathbf{r}\in\overline{V}=V\cup B$,

(b) $(\nabla^2+k^2)v(\mathbf{r})=0$, $\mathbf{r}\in\overline{V}$; k, the wave number, is assumed to be complex,

(c) $r(\partial v/\partial r-ikv)=o(1)$, $r\to\infty$, uniformly in θ and ϕ.

Then, we obtain the equation

$$\omega = K_1\omega + \overset{(0)}{u_1} \tag{D}$$

for the Dirichlet problem, and the equation

$$\omega = (K_1 + K_2)\omega + \overset{(0)}{u_2} \tag{N}$$

for the Neumann problem; where

$$\omega = \exp(-ikr)v, \tag{1}$$

$$\omega \to K_1\omega = -2ik\int_V dv_1 \frac{G_0(\mathbf{r}, \mathbf{r}_1)}{r_1}\frac{\partial}{\partial r}[r_1\omega(\mathbf{r}_1)], \tag{2}$$

$$\omega \to K_2\omega = ik\int_B d\sigma_B G_0(\mathbf{r}, \mathbf{r}_B)\hat{n}\circ\hat{\mathbf{r}}_B\omega(\mathbf{r}_B), \tag{3}$$

$$\overset{(0)}{u_1} = \int_B d\sigma_B\omega(\mathbf{r}_B)\frac{\partial}{\partial n}G_0(\mathbf{r}, \mathbf{r}_B), \tag{4}$$

$$\overset{(0)}{u_2} = -\int_B d\sigma_B G_0(\mathbf{r}, \mathbf{r}_B)\exp(-ikr_B)\frac{\partial\omega(\mathbf{r}_B)}{\partial n}\cdot \tag{5}$$

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