A NONLINEAR STURM-LIOUVILLE PROBLEM[1]

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We consider the following nonlinear boundary value problems on the interval $\alpha \leqq x \leqq \beta$:

$$[p(x)y']' + q(x)y + \lambda y[a(x) \pm h(x, y, y')] = 0, \tag{$1^+, 1^-$}$$

$$y(\alpha) + \gamma_1 y'(\alpha) = 0, \qquad y(\beta) + \gamma_2 y'(\beta) = 0. \tag{2}$$

Here λ is a real constant. We assume that $p(x) > 0$, $a(x) > 0$, $p'(x)$, and $q(x)$ are continuous in $\alpha \leqq x \leqq \beta$. In addition in the region

$$D = \{(x, y, z) \mid \alpha \leqq x \leqq \beta, \ -\infty < y < \infty, \ -\infty < z < \infty\},$$

we require h to satisfy the following conditions:

$$h(x, y, z) \text{ is defined and continuous,} \tag{3}$$

$$h(x, y, z) \geqq 0, \tag{4}$$

$$h(x, 0, 0) = 0, \tag{5}$$

$$\lim_{c \to +\infty} h(x, c\xi, c\eta) = \infty \text{ uniformly for } \alpha \leqq x \leqq \beta \tag{6}$$

and for all $\xi \neq 0$, $\eta \neq 0$.

We also assume that γ_1, γ_2, and $q(x)$ are such that the eigenvalues, λ_n, of the "linearized" problem,

$$[p(x)\mathcal{Y}_n']' + q(x)\mathcal{Y}_n + \lambda_n a(x)\mathcal{Y}_n = 0 \tag{7}$$

with boundary conditions (2), satisfy

$$0 < \lambda_1 < \lambda_2 < \cdots < \lambda_k < \cdots, \tag{8}$$

with $\mathcal{Y}_k$ having $k-1$ zeros in the open interval (α, β).

Previous studies of special cases of equations (1^+) and (1^-) with boundary conditions (2) have been done by Ljusternik [4], Nehari [5] and Pimbley [6], [7] among others. Similar nonlinear eigenvalue problems for partial differential equations have been treated by Berger [1], Browder [2], and Levinson [3].

We treat the question of the existence and multiplicity of solutions of equations (1^+) and (1^-) with boundary conditions (2). There are

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