ON THE HAUPTVERMUTUNG FOR MANIFOLDS

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The "Hauptvermutung" is the conjecture that homeomorphic (finite) simplicial complexes have isomorphic subdivisions, i.e. homeomorphic implies piecewise linearly homeomorphic. It was formulated in the first decade of this century and seems to have been inspired by the question of the topological invariance of the Betti and torsion numbers of a finite simplicial complex.

The Hauptvermutung is known to be true for simplicial complexes of dimension <4,[1] but there are counterexamples in each dimension >4 (Milnor, 1961).

The Milnor examples, K and L, have two notable properties:

(i) K and L are not manifolds,

(ii) K and L are not locally isomorphic.

Thus it is natural to restrict the Hauptvermutung to the class of piecewise linear n-manifolds, simplicial complexes where each point has a neighborhood which is piecewise linearly homeomorphic to Euclidean space R^n or Euclidean half space R^n_+.

We assume that $H_3(M, Z)$ has no 2-torsion.[2]

THE MAIN THEOREM. *Let h be a homeomorphism between compact PL-manifolds L and M. Then for some integer p*

$$(L, \partial L) \times R^p \xrightarrow{h \times \text{identity}} (M, \partial M) \times R^p$$

is properly homotopic to a PL-homeomorphism. If dim $M \geqq 6$ *and* $\pi_1 M = \pi_1 \partial M = 0$, *then h is homotopic to a PL-homeomorphism.*

There are three steps in the proof of the Main Theorem.

For simplicity we assume now that M and L are closed simply connected PL-manifolds of dim >4.

DEFINITION 1. Let g: $L \to M$ be a homotopy equivalence. A *triangulation of g* is a homotopy of g to a PL-homeomorphism.

[1] 1-complexes, obvious. 2-manifolds–Rado, 1926. 2-complexes–Papakyriakopoulos, 1943. 3-manifolds–Moise, 1953. 3-complexes–E. Brown, 1964.

[2] We state this condition in terms of H_3 instead of H^4 to suggest a connection with the three dimensional Poincare conjecture.