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WEAK CONVERGENCE OF THE SEQUENCE OF SUCCESSIVE APPROXIMATIONS FOR NONEXPANSIVE MAPPINGS

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In a recent paper [**4**] F. E. Browder and W. V. Petryshyn have shown that if a nonexpansive mapping $T: X\to X$ of a Hilbert space X into itself is asymptotically regular and has at least one fixed point then, for any x in X, a weak limit of a weakly convergent subsequence of the sequence of successive approximations $\{T^n x\}$ is a fixed point of T. The main object of the present note is to strengthen considerably this result by showing that under the same assumptions the sequence $\{T^n x\}$ is necessarily weakly convergent.

In §1 we recall some basic definitions and prove two simple lemmas. In §2 we prove the weak convergence of the sequence $\{T^n x\}$ and in §3 we discuss the possibility of the extension of this result to Banach spaces having weakly continuous duality mappings. In §4 an application of Theorem 2 stated in §3 to a modified sequence of successive approximations is given and, in §5, limits of validity of the first key lemma of §1 are discussed.

1. Let C be a convex closed set in a Banach space X. A mapping T: $C\to X$ is called *nonexpansive* if $\|Tx-Ty\|\leqq\|x-y\|$ for any x, y in C. Following [**4**], a mapping T: $C\to C$ is said to be *asymptotically regular* if, for any x in C, the sequence $\{T^{n+1}x-T^n x\}=\{(I-T)(T^n x)\}$ tends to zero as $n\to\infty$. Finally, a mapping T: $C\to X$ is called *demiclosed* if its graph in $C\times X$ is closed in the topology of a Cartesian product induced in $C\times X$ by the weak topology in C and the strong topology in X; i.e., if for any sequence $\{x_n\}\subset C$ which converges weakly to an x_0 in C, the strong convergence of the sequence $\{Tx_n\}$ to a y_0 in X implies that $Tx_0=y_0$.