# THE DIFFEOMORPHISM GROUP OF A COMPACT RIEMANN SURFACE

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1. **Introduction.** In this note we announce two theorems. The first describes the homotopy type of the topological group $\mathfrak{D}(X)$ of diffeomorphisms ($=C^\infty$-diffeomorphisms) of a compact oriented surface $X$ without boundary. The second, of which the first is a corollary, gives a fundamental relation among $\mathfrak{D}(X)$, the space of complex structures on $X$, and the Teichmüller space $T(X)$ of $X$. We make essential use of the theory of quasiconformal mappings and Teichmüller spaces developed by Ahlfors and Bers [**3**], [**6**], and the theory of fibrations of function spaces. Our results confirm a conjecture of Grothendieck [**7**, p. 7-09], relating the homotopy of $\mathfrak{D}(X)$ and $T(X)$.

2. **The theorems.** The surface $X$ has a unique (up to equivalence) $C^\infty$-differential structure. Let $\mathfrak{D}(X)$ denote the group of orientation preserving diffeomorphisms. With the $C^\infty$-topology (uniform convergence of all differentials) $\mathfrak{D}(X)$ is a metrizable topological group [**8**]. We let $\mathfrak{D}_0(X; x_1, \cdots, x_n)$ denote the subgroup of $\mathfrak{D}(X)$ consisting of those diffeomorphisms $f$ which are homotopic to the identity and satisfy $f(x_i)=x_i$ $(1\leqq i\leqq n)$, where $x_1, \cdots, x_n$ are distinct points of $X$. This second condition is fulfilled vacuously if $n=0$.

THEOREM 1. *Let $g$ denote the genus of $X$.*

(a) *If $g=0$, then $\mathfrak{D}_0(X; x_1, x_2, x_3)$ is contractible. Furthermore, $\mathfrak{D}(X)$ is homeomorphic to $G\times\mathfrak{D}_0(X; x_1, x_2, x_3)$, where $G$ is the group of conformal automorphisms of the Riemann sphere.*

(b) *If $g=1$, then $\mathfrak{D}_0(X; x_1)$ is contractible. Furthermore, $\mathfrak{D}_0(X)$ is homeomorphic to $G\times\mathfrak{D}_0(X; x_1)$, where now $G$ is the identity component of the group of conformal automorphisms of the torus.*

(c) *If $g\geqq 2$, then $\mathfrak{D}_0(X)$ is contractible.*

COROLLARY. *In all cases $\mathfrak{D}_0(X)$ is the identity component of $\mathfrak{D}(X)$.*

REMARK 1. Part (a) is equivalent to the theorem of Smale [**9**] asserting that the rotation group SO(3) is a strong deformation retract of $\mathfrak{D}(S^2)$. Our proof is entirely different from Smale's.

REMARK 2. A concept of differentiability has recently been de-

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